ANNOUNCEMENTS

• A summary of frequently misunderstood/missed concepts is now posted on the class website, and will be updated regularly.
• Graded HW assignments can be picked up in lab (353 Cory).
  → Please indicate your lab section on your HW assignments!
• Midterm #1 Review: 3-5PM in 306 Soda – will be archived online

OUTLINE

• BJT Amplifiers (cont’d)
  – Common-emitter topology
    – CE stage with emitter degeneration
    – Impact of Early effect ($r_o$)

Reading: Finish Chapter 5.3.1
Emitter Degeneration

- By inserting a resistor in series with the emitter, we “degenerate” the CE stage.
- This topology will decrease the gain of the amplifier but improve other aspects, such as linearity, and input impedance.

![Degenerated CE stage diagrams](a) and (b)
Small-Signal Analysis

- The gain of a degenerated CE stage = the total load resistance seen at the collector divided by \(1/g_m\) plus the total resistance placed in series with the emitter.

\[
A_v = \frac{-g_m R_C}{1 + g_m R_E} = \frac{-R_C}{1 + \frac{1}{g_m} + R_E}
\]
Emitter Degeneration Example 1

Note that the input impedance of $Q_2$ is in parallel with $R_E$.

$$A_v = -\frac{R_C}{\frac{1}{g_m} + R_E || r_{\pi 2}}$$
Emitter Degeneration Example 2

Note that the input impedance of $Q_2$ is in parallel with $R_C$.

\[ A_v = - \frac{R_C \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E} \]
Input Impedance of Degenerated CE Stage

- With emitter degeneration, the input impedance is increased from $r_\pi$ to $r_\pi + (\beta+1)R_E$ — a desirable effect.

\[(V_A = \infty)\]

\[v_x = r_\pi i_x + R_E (1 + \beta)i_x\]

\[R_{in} \equiv \frac{v_x}{i_x} = r_\pi + (\beta+1)R_E\]
Output Impedance of Degenerated CE Stage

- Emitter degeneration does not alter the output impedance, if the Early effect is negligible.

\[
\begin{align*}
V_i &= 0 = V_\pi + \left( \frac{V_\pi}{r_\pi} + g_m V_\pi \right) R_E 
\Rightarrow V_\pi &= 0
\end{align*}
\]

\[
R_{out} \equiv \frac{V_x}{i_x} = R_C
\]
Degenerated CE Stage as a “Black Box”

\[ V_A = \infty \]
\[ i_{out} = g_m v_{in} \]
\[ i_{out} = g_m \frac{v_{in}}{1 + (r_\pi^{-1} + g_m)R_E} \]

\[ G_m \equiv \frac{i_{out}}{v_{in}} \approx \frac{g_m}{1 + g_m R_E} \]

- If \( g_m R_E >> 1 \), \( G_m \) is more linear.
Degenerated CE Stage with Base Resistance

\( V_A = \infty \)

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{V_A}{V_{\text{in}}} \cdot \frac{V_{\text{out}}}{V_A}
\]

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-\beta R_C}{r_\pi + (\beta + 1)R_E + R_B}
\]

\[
A_v \approx \frac{-R_C}{1 + \frac{R_B}{g_m R_E + \beta + 1}}
\]

\[
\frac{V_A}{V_{\text{in}}} = \frac{r_\pi + (\beta + 1)R_E}{R_B + \frac{1}{r_\pi} + (\beta + 1)R_E}
\]

\[
\frac{V_{\text{out}}}{V_A} = \frac{-g_m R_C}{1 + \left(\frac{1}{r_\pi} + g_m\right)R_E} \cdot \frac{r_\pi}{r_\pi}
\]
Degenerated CE Stage: Input/Output Impedances

- $R_{in1}$ is more important in practice, because $R_B$ is often the output impedance of the previous stage.

\[
(V_A = \infty) \\
R_{in1} = r_\pi + (\beta + 1)R_E \\
R_{in2} = R_B + r_\pi + (\beta + 1)R_E \\
R_{out} = R_C
\]
Emitter Degeneration Example 3

\[ A_v = \frac{1}{ \frac{1}{g_m} R_2 + \frac{R_B}{\beta + 1} \parallel (R_C \parallel R_1) } \]

\[ R_{in} = r_\pi + (\beta + 1)R_2 + R_B \]

\[ R_{out} = R_C \parallel R_1 \]
Output Impedance of Degenerated CE Stage with $V_A < \infty$

- Emitter degeneration boosts the output impedance.
  - This improves the gain of the amplifier and makes the circuit a better current source.

\[
\frac{V_{out}}{i_X} \equiv R_{out} = \left[ 1 + g_m \left( \frac{R_E}{r_\pi} \right) \right] r_O + \frac{R_E}{r_\pi}
\]

\[
R_{out} = r_O + \left( g_m r_O + 1 \right) \left( \frac{R_E}{r_\pi} \right)
\]

\[
R_{out} \approx r_O \left[ 1 + g_m \left( \frac{R_E}{r_\pi} \right) \right]
\]
Two Special Cases

Stage with explicit depiction of $r_o$:

1) $R_E >> r_\pi : R_{out} \approx r_O (1 + g_m r_\pi) \approx \beta r_O$

2) $R_E << r_\pi : R_{out} \approx (1 + g_m R_E) r_O$
Analysis by Inspection

• This seemingly complicated circuit can be greatly simplified by first recognizing that the capacitor creates an AC short to ground, and gradually transforming the circuit to a known topology.

\[
R_{out} = R_1 \parallel R_{out1} \quad R_{out1} = [1 + g_m (R_2 \parallel r_\pi)]r_O \quad R_{out} = [1 + g_m (R_2 \parallel r_\pi)]r_O \parallel R_1
\]
Example: Degeneration by Another BJT

\[ R_{out} = \left[ 1 + g_{m1} \left( r_{O2} \parallel r_{\pi 1} \right) \right] r_{O1} \]

- Called a “cascode”, this circuit offers many advantages that we will study later...
Bad Input Connection

- Since the microphone has a very low resistance (connecting the base of $Q_1$ to ground), it attenuates the base voltage and renders $Q_1$ with a very small bias current.
Use of Coupling Capacitor

- A capacitor is used to isolate the DC bias network from the microphone, and to short (or “couple”) the microphone to the amplifier at higher frequencies.
DC and AC Analysis

- The coupling capacitor is replaced with an open circuit for DC analysis, and then replaced with a short circuit for AC analysis.

\[
A_v = -g_m (R_C \parallel r_O)
\]

\[
R_{in} = r_\pi \parallel R_B
\]

\[
R_{out} = R_C \parallel r_O
\]
Bad Output Connection

- Since the speaker has an inductor with very low DC resistance, connecting it directly to the amplifier would ~short the collector to ground, causing the BJT to go into deep saturation mode.
Use of Coupling Capacitor at Output

- The AC coupling indeed allows for correct biasing. However, due to the speaker’s small input impedance, the overall gain drops considerably.

\[ |A_v| = g_m \left( \frac{R_c}{R_{sp}} \right) \]
CE Stage with Voltage-Divider Biasing

\[
A_v = -g_m (R_C \parallel r_O)
\]

\[
R_{in} = r_\pi \parallel R_1 \parallel R_2
\]

\[
R_{out} = R_C \parallel r_O
\]
CE Stage with Robust Biasing

\[ V_A = \infty \]

\[ A_v = \frac{-R_C}{1 + R_E g_m} \]

\[ R_{in} = [r_s + (\beta + 1)R_E] || R_1 || R_2 \]

\[ R_{out} = R_C \]
Elimination of Emitter Degeneration for AC Signals

- The capacitor $C_2$ shorts out $R_E$ at higher frequencies to eliminate the emitter degeneration.

\[ (V_A = \infty) \]

\[ A_v = -g_m R_C \]

\[ R_{in} = r_\pi \parallel R_1 \parallel R_2 \]

\[ R_{out} = R_C \]
Complete CE Stage

\[ A_v = \frac{-R_C \parallel R_L}{1 + R_E + \frac{R_s \parallel R_1 \parallel R_2}{\beta + 1}} \cdot \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_s} \]
Summary of CE Concepts

\[ A_v = -g_m R_C \]

Gain

\[ A_v = -g_m (R_C \parallel r_O) \]

\[ A_v, R_{in} \]

Headroom

\[ R_{in}, R_{out} \]

\[ R_{out} \]

\[ R_E \]

\[ R_1, R_2, C_1, C_2 \]