ANNOUNCEMENTS

• HW1 will be considered as extra credit.
• HW3 will be posted tonight, due Tuesday 9/18.
• Monday 3PM (247 Cory) discussion section has room!

OUTLINE

• BJT (cont’d)
  – Transconductance
  – Small-signal model
  – The Early effect
  – BJT operation in saturation mode

Reading: Chapter 4.4.3-4.5
Notes on PN Junctions

• Typically, pn junctions in IC devices are formed by counter-doping. The equations provided in class (and in the textbook) can be readily applied to such diodes if
  - \( N_A \equiv \text{net acceptor doping on p-side} \ \ (N_A - N_D)_{p-side} \)
  - \( N_D \equiv \text{net donor doping on n-side} \ \ (N_D - N_A)_{n-side} \)

\[ I_D = I_S \left( e^{qV_D/kT} - 1 \right) \]

\[ I_S = A q n_i^2 \left( \frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right) \]
Transconductance, $g_m$

- The **transconductance** ($g_m$) of a transistor is a measure of how well it converts a voltage signal into a current signal.
- It will be shown later that $g_m$ is one of the most important parameters in integrated circuit design.

\[
g_m \equiv \frac{dI_C}{dV_{BE}} \approx \frac{d}{dV_{BE}} \left( I_s \exp \frac{V_{BE}}{V_T} \right)
\]

\[
g_m = \frac{1}{V_T} I_s \exp \frac{V_{BE}}{V_T}
\]

\[
g_m = \frac{I_C}{V_T}
\]

\[
V_T \approx \frac{kT}{e}
\]
Visualization of Transconductance

- $g_m$ can be visualized as the slope of the $I_C$ vs. $V_{BE}$ curve.
- The slope (hence $g_m$) increases with $I_C$. 

![Graph showing the relationship between $I_C$, $V_{BE}$, and $g_m$]
Transconductance and $I_C$

- For a given $V_{BE}$ swing ($\Delta V$), the resulting current swing about $I_{C2}$ is larger than it is about $I_{C1}$.
  - This is because $g_m$ is larger when $V_{BE} = V_{B2}$.

\[ V_{BE} = V_{B2} + \Delta V \]
\[ V_{BE} = V_{B2} \]
\[ V_{BE} = V_{B1} + \Delta V \]
\[ V_{BE} = V_{B1} \]
Transconductance and Emitter Area

- When the BJT emitter area is increased by a factor $n$, $I_S$ increases by the factor $n$.
- For a fixed value of $V_{BE}$, $I_C$ and hence $g_m$ increase by a factor of $n$.
Derivation of Small-Signal Model

- The BJT small-signal model is derived by perturbing the voltage difference between two terminals while fixing the voltage on the third terminal, and analyzing the resultant changes in terminal currents.
  - This is done for each of the three terminals as the one with fixed voltage.
  - We model the current change by a controlled source or resistor.

\[\Delta V\]
\[\Delta I_B\]
\[\Delta I_C\]
\[\Delta I_E\]
\[V_{CE}\]

\[V_{BE}\]
\[\Delta I_C\]
\[\Delta I_B\]
\[\Delta I_E\]
\[\Delta V\]
Small-Signal Model: $V_{BE}$ Change

$V_{CE}$ is fixed

$\Delta I_C = g_m \Delta V_{BE}$

$\Delta V_{BE} = g_m \Delta V_{BE}$

$\Delta I_B \rightarrow B$

$\Delta I_C \rightarrow C$

$R_{\pi} = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{1}{\beta g_m}$

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Small-Signal Model: $V_{CE}$ Change

• Ideally, $V_{CE}$ has no effect on the collector current. Thus, it will not contribute to the small-signal model.

• It can be shown that $V_{CB}$ ideally has no effect on the small-signal model, either.
Small-Signal Model: Example 1

- The small-signal model parameters are calculated for the DC operating point, and are used to determine the change in $I_C$ due to a change in $V_{BE}$.

\[ I_S = 3 \times 10^{-16} \text{ A} \]
\[ \beta = 100 \]
\[ I_C = I_S \exp \left( \frac{V_{BE}}{\beta K T} \right) \]
\[ I_C = 6.92 \text{ mA} \]

\[ V_{BE} = 0.8 \text{ V}, \quad V_T = 26 \text{ mV} \]

\[ g_m = \frac{I_C}{V_T} = \frac{1}{3.75 \Omega} \]
\[ r_\pi = \frac{\beta}{g_m} = 375 \Omega \]

\[ \left[ \frac{1}{\Omega} \equiv \text{Siemens} \right] \]
Small-Signal Model: Example 2

• In this example, a resistor is placed between the power supply and collector, to obtain an output voltage signal.

• Since the power supply voltage does not vary with time, it is regarded as ground (reference potential) in small-signal analysis.

\[ V_{cc} + V_{cc} = V_{out} \]

\[ V_{out} = \frac{1}{1 + g_m r_{\pi}} \]

\[ I_{cc} = I_{out} \]

\[ V_{cc} = 1.8 \text{ V} \]

\[ R_C = 100 \ \Omega \]

\[ V_1 = 800 \text{ mV} \]
The Early Effect

• In reality, the collector current depends on $V_{CE}$:
  
  – For a fixed value of $V_{BE}$, as $V_{CE}$ increases, the reverse bias on the collector-base junction increases, hence the width of the depletion region increases. Therefore, the quasi-neutral base width decreases, so that collector current increases.

To minimize this effect: dope base more heavily than collector.
Early Effect: Impact on BJT $I-V$

- Due to the Early effect, collector current increases with increasing $V_{CE}$, for a fixed value of $V_{BE}$. 

\[ I_C = I_S \exp \left( \frac{V_{BE1}}{V_T} \right) \]
Early Effect Representation

\[ V_A = \text{Early Voltage} \]

\[ (I_S \exp \frac{V_1}{V_T}) \left(1 + \frac{V_X}{V_A} \right) \]
The Early effect can be accounted for, by simply multiplying the collector current by a correction factor.

The base current does not change significantly.
Early Effect and Small-Signal Model

\[ r_o \equiv \frac{\Delta V_{CE}}{\Delta I_C} = \frac{V_A}{I_S \exp \frac{V_{BE}}{V_T}} \approx \frac{V_A}{I_C} \]
Summary of BJT Concepts

- **Operation in Active Mode**
  - Reverse biased
  - Forward biased

- **Large-Signal Model**
  - \( I_C = I_S \exp \left( \frac{V_{BE}}{V_T} \right) \)

- **I/V Characteristics**
  - \( I_C = I_S \exp \left( \frac{V_{BE1}}{V_T} \right) \)

- **Small-Signal Model**
  - \( I_C = I_S \exp \left( \frac{V_{BE}}{V_T} \right) \)

- **Early Effect**
  - \( V_{CE} \)

- **Modified Small-Signal Model**
  - \( I_C = \beta \cdot I_S \exp \left( \frac{V_{BE}}{V_T} \right) \)

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BJT in Saturation Mode

- When the collector voltage drops below the base voltage, the collector-base junction is forward biased. Base current increases, so that the current gain ($I_C/I_B$) decreases.
Large-Signal Model for Saturation Mode

\[ I_{S2} \exp \frac{V_{BE}}{V_T} \]

\[ D_{BC} \]

\[ V_{DE} \]

\[ n_{BE} \]

\[ I_{S1} \exp \frac{V_{BE}}{V_T} \]

\[ V_{BE} \]

\[ V_{DE} \]

\[ D_{BE} \]

\[ I_{S1} \exp \frac{V_{BE}}{V_T} \]
BJT Output Characteristics

- The operating speed of the BJT also drops in saturation.
Example: Acceptable $V_{CC}$ Range

- In order to prevent the BJT from entering very deeply into saturation, the collector voltage must not fall below the base voltage by more than 400 mV.

\[
V_{CC} \geq I_C R_C + (V_{BE} - 400 \text{ mV})
\]
Deep Saturation

• In deep saturation, the BJT does not behave as a voltage-controlled current source.
• $V_{CE}$ is relatively constant.