Lecture 5

ANNOUNCEMENTS
• HW1 will be considered as extra credit.
• HW3 is posted, due Tuesday 9/18

OUTLINE
• BJT (cont’d)
  – Transconductance
  – Small-signal model
  – The Early effect
  – BJT operation in saturation mode

Reading: Chapter 4.4.3-4.5
Transconductance, $g_m$

- The **transconductance** ($g_m$) of a transistor is a measure of how well it converts a voltage signal into a current signal.
- It will be shown later that $g_m$ is one of the most important parameters in integrated circuit design.

\[
g_m \equiv \frac{dI_C}{dV_{BE}} \approx \frac{d}{dV_{BE}} \left( I_s \exp \frac{V_{BE}}{V_T} \right)
\]

\[
g_m = \frac{1}{V_T} I_s \exp \frac{V_{BE}}{V_T}
\]

\[
g_m = \frac{I_C}{V_T}
\]
Visualization of Transconductance

- $g_m$ can be visualized as the slope of the $I_C$ vs. $V_{BE}$ curve.
- The slope (hence $g_m$) increases with $I_C$. 

![Diagram showing $I_C$ vs. $V_{BE}$ with $g_m \Delta V$ and $\Delta V$ indicating the slope and the change in voltage respectively.](image)
Transconductance and $I_C$

- For a given $V_{BE}$ swing ($\Delta V$), the resulting current swing about $I_{C2}$ is larger than it is about $I_{C1}$.
  - This is because $g_m$ is larger when $V_{BE} = V_{B2}$.
Transconductance and Emitter Area

- When the BJT emitter area is increased by a factor $n$, $I_S$ increases by the factor $n$.

→ For a fixed value of $V_{BE}$, $I_C$ and hence $g_m$ increase by a factor of $n$. 

![Diagram](image.png)
Derivation of Small-Signal Model

• The BJT small-signal model is derived by perturbing the voltage difference between two terminals while fixing the voltage on the third terminal, and analyzing the resultant change in current.
  – This is done for each of the three terminals as the one with fixed voltage.
  – We model the current change by a controlled source or resistor.
Small-Signal Model: $V_{BE}$ Change

\[ \Delta I_C = g_m \Delta V_{BE} \]

\[ \Delta I_C = I_s \exp \left( \frac{V_{BE} + \Delta V_{BE}}{V_T} \right) \]

\[ \Delta V_{BE} = g_m \Delta V_{BE} \]

\[ \Delta V_{BE} = \pi \]

\[ \Delta V_{BE} = g_m \pi \]

\[ \Delta V_{BE} = r_{\pi} \]

\[ \Delta V_{BE} = g_m \pi \]
Small-Signal Model: $V_{CE}$ Change

- Ideally, $V_{CE}$ has no effect on the collector current. Thus, it will not contribute to the small-signal model.
- It can be shown that $V_{CB}$ ideally has no effect on the small-signal model, either.
The small-signal model parameters are calculated for the DC operating point, and are used to determine the change in \( I_C \) due to a change in \( V_{BE} \).

\[
g_m = \frac{I_C}{V_T} = \frac{1}{3.75 \Omega}
\]

\[
r_\pi = \frac{\beta}{g_m} = 375 \Omega
\]
Small-Signal Model: Example 2

- In this example, a resistor is placed between the power supply and collector, to obtain an output voltage signal.

- Since the power supply voltage does not vary with time, it is regarded as ground (reference potential) in small-signal analysis.
The Early Effect

- In reality, the collector current depends on $V_{CE}$:
  - For a fixed value of $V_{BE}$, as $V_{CE}$ increases, the reverse bias on the collector-base junction increases, hence the width of the depletion region increases. Therefore, the quasi-neutral base width decreases, so that collector current increases.
Early Effect: Impact on BJT $I-V$

- Due to the Early effect, collector current increases with increasing $V_{CE}$, for a fixed value of $V_{BE}$.

\[ I_C = I_S \exp \left( \frac{V_{BE_1}}{V_T} \right) \]
Early Effect Representation

\[ I_S \exp \left( \frac{V_1}{V_T} \right) \left( 1 + \frac{V_X}{V_A} \right) \]
The Early effect can be accounted for, by simply multiplying the collector current by a correction factor. The base current does not change significantly.
Early Effect and Small-Signal Model

\[ r_o \equiv \frac{\Delta V_{CE}}{\Delta I_C} = \frac{V_A}{I_s \exp \frac{V_{BE}}{V_T}} \approx \frac{V_A}{I_C} \]
Summary of BJT Concepts

Operation in Active Mode

Large-Signal Model

I/V Characteristics

Small-Signal Model

Early Effect

Modified Small-Signal Model
BJT in Saturation Mode

- When the collector voltage drops below the base voltage, the collector-base junction is forward biased. Base current increases, so that the current gain \((I_C/I_B)\) decreases.
Large-Signal Model for Saturation Mode

![Diagrams](image-url)
BJT Output Characteristics

- The operating speed of the BJT also drops in saturation.
Example: Acceptable $V_{CC}$ Range

- In order to prevent the BJT from entering very deeply into saturation, the collector voltage must not fall below the base voltage by more than 400 mV.

\[ V_{CC} \geq I_C R_C + (V_{BE} - 400 \text{ mV}) \]
Deep Saturation

- In deep saturation, the BJT does not behave as a voltage-controlled current source.
- $V_{CE}$ is $\sim$constant.