Announcements

- HW1 is posted, due Tuesday 9/4
- Discussion Section 102 (We 9-10) moved to 289 Cory
- TA office hrs will be held in 353 Cory until further notice.
- Lab sections:
  - The Wed. PM section is over-subscribed! The following students will likely need to sign up for an alternative section:
    - Chang, Phillip
    - Cockrell, Matthew
    - Dwan, Kevin
    - Li, Xue Cong
    - Liu, Guan Quan
    - Park, Manjae
    - Toriyama, Yuta
    - Tran, Anthony
  - Final lab section assignments will be posted online on 9/1.
- Paid position for videotaping EE105 lectures is available
OUTLINE

• Basic Semiconductor Physics (cont’d)
  – Carrier drift and diffusion

• PN Junction Diodes
  – Electrostatics
  – Capacitance

Reading: Chapter 2.1-2.2
Dopant Compensation

- An N-type semiconductor can be converted into P-type material by counter-doping it with acceptors such that $N_A > N_D$.

- A **compensated semiconductor material** has both acceptors and donors.

<table>
<thead>
<tr>
<th>N-type material</th>
<th>P-type material</th>
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<tr>
<td>$(N_D &gt; N_A)$</td>
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<tr>
<td>$n \approx N_D - N_A$</td>
<td>$p \approx N_A - N_D$</td>
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<tr>
<td>$p \approx \frac{n_i^2}{N_D - N_A}$</td>
<td>$n \approx \frac{n_i^2}{N_A - N_D}$</td>
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Types of Charge in a Semiconductor

• Negative charges:
  – Conduction electrons (density = $n$)
  – Ionized donor atoms (density = $N_D$)

• Positive charges:
  – Holes (density = $p$)
  – Ionized acceptor atoms (density = $N_A$)

• The net charge density ($C/cm^3$) in a semiconductor is

$$\rho = q(p - n + N_D - N_A)$$
Carrier Drift

- The process in which charged particles move because of an electric field is called \textit{drift}.
- Charged particles within a semiconductor move with an average velocity proportional to the electric field.
  - The proportionality constant is the carrier \textit{mobility}.

\[ F = qE \]

\[ \vec{E} = \mu \vec{E} \]

\[ \vec{v}_h = \mu_p \vec{E} \]

\[ \vec{v}_e = -\mu_n \vec{E} \]

**Notation:**
- $\mu_p \equiv$ hole mobility (cm$^2$/V·s)
- $\mu_n \equiv$ electron mobility (cm$^2$/V·s)
Velocity Saturation

• In reality, carrier velocities saturate at an upper limit, called the *saturation velocity* \( v_{sat} \).

\[
\begin{align*}
\mu &= \frac{\mu_0}{1+bE} \\
\nu_{sat} &= \frac{\mu_0}{b} \\
\nu &= \frac{\mu_0 E}{1+\frac{\mu_0 E}{\nu_{sat}}} 
\end{align*}
\]
Drift Current

- Drift current is proportional to the carrier velocity and carrier concentration:

\[ v_h \ t \ A = \text{volume from which all holes cross plane in time } t \]

\[ p \ v_h \ t \ A = \# \text{ of holes crossing plane in time } t \]

\[ q \ p \ v_h \ t \ A = \text{charge crossing plane in time } t \]

\[ q \ p \ v_h \ A = \text{charge crossing plane per unit time = hole current} \]

⇒ Hole current per unit area (i.e. current density) \[ J_{p,\text{drift}} = q \ p \ v_h \]
Conductivity and Resistivity

• In a semiconductor, both electrons and holes conduct current:

\[
J_{p,\text{drift}} = qp \mu_p E \quad J_{n,\text{drift}} = -qn(-\mu_n E)
\]

\[
J_{\text{tot,drift}} = J_{p,\text{drift}} + J_{n,\text{drift}} = qp \mu_p E + qn \mu_n E
\]

\[
J_{\text{tot,drift}} = q(p\mu_p + n\mu_n)E \equiv \sigma E
\]

• The **conductivity** of a semiconductor is \( \sigma \equiv qp \mu_p + qn \mu_n \)
  – Unit: mho/cm

• The **resistivity** of a semiconductor is \( \rho \equiv \frac{1}{\sigma} \)
  – Unit: ohm-cm

  Typical range for Si: \(10^3 \text{\Omega} \cdot \text{cm} \leq 10^{-3} \text{\Omega} \cdot \text{cm}\)
Resistivity Example

- Estimate the resistivity of a Si sample doped with phosphorus to a concentration of $10^{15}$ cm$^{-3}$ and boron to a concentration of $10^{17}$ cm$^{-3}$. The electron mobility and hole mobility are 800 cm$^2$/Vs and 300 cm$^2$/Vs, respectively.

\[ q = 1.6 \times 10^{-19} \text{ C} \]

\[ N_D = 10^{15} \text{ cm}^{-3} \]
\[ N_A = 10^{17} \text{ cm}^{-3} \]

\[ N_A > N_D \Rightarrow p \rightarrow p \epsilon \]

\[ \rho = N_A - N_D = 10^{17} \text{ cm}^{-3} \]
\[ n = 10^3 \text{ cm}^{-3} \]

\[ \rho = \left( \mu_n p + \frac{q \mu_p n}{1.6 \times 10^{-19} \times 300 \times 10^{17}} \right)^{-1} \approx 0.2 \text{ S} \cdot \text{cm} \]
Electrical Resistance

where $\rho$ is the resistivity

Resistance $R \equiv \frac{V}{I} = \rho \frac{L}{Wt}$ (Unit: ohms)
Carrier Diffusion

- Due to thermally induced random motion, mobile particles tend to move from a region of high concentration to a region of low concentration.
  - Analogy: ink droplet in water
- Current flow due to mobile charge diffusion is proportional to the carrier concentration gradient.
  - The proportionality constant is the diffusion constant.

\[ J_p = -qD_p \frac{dp}{dx} \]

**Notation:**
- \( D_p \equiv \) hole diffusion constant (cm\(^2\)/s)
- \( D_n \equiv \) electron diffusion constant (cm\(^2\)/s)
Diffusion Examples

- Linear concentration profile ➔ constant diffusion current
  \[ p = N \left( 1 - \frac{x}{L} \right) \]
  \[ J_{p, \text{diff}} = -qD_p \frac{dp}{dx} = qD_p \frac{N}{L} \]

- Non-linear concentration profile ➔ varying diffusion current
  \[ p = N \exp \left( -\frac{x}{L_d} \right) \]
  \[ J_{p, \text{diff}} = -qD_p \frac{dp}{dx} = \frac{qD_p N}{L_d} \exp \left( -\frac{x}{L_d} \right) \]
Diffusion Current

- Diffusion current within a semiconductor consists of hole and electron components:

\[ J_{p,\text{diff}} = -qD_p \frac{dp}{dx} \quad J_{n,\text{diff}} = qD_n \frac{dn}{dx} \]

\[ J_{\text{tot,\text{diff}}} = q(D_n \frac{dn}{dx} - D_p \frac{dp}{dx}) \]

- The total current flowing in a semiconductor is the sum of drift current and diffusion current:

\[ J_{\text{tot}} = J_{p,\text{drift}} + J_{n,\text{drift}} + J_{p,\text{diff}} + J_{n,\text{diff}} \]
The Einstein Relation

- The characteristic constants for drift and diffusion are related:

\[ \frac{D}{\mu} = \frac{kT}{q} \]

- Note that \( \frac{kT}{q} \approx 26\text{mV} \) at room temperature (300K)
  - This is often referred to as the “thermal voltage”.
The PN Junction Diode

- When a P-type semiconductor region and an N-type semiconductor region are in contact, a PN junction diode is formed.

(a) Silicon atoms with P and B dopants

(b) Schematic of a PN junction diode with terminals labeled Cathode and Anode.
Diode Operating Regions

- In order to understand the operation of a diode, it is necessary to study its behavior in three operation regions: equilibrium, reverse bias, and forward bias.

\[ V_D = 0 \]

- PN Junction in Equilibrium
  - Depletion Region
  - Built-in Potential

\[ V_D < 0 \]

- PN Junction Under Reverse Bias
  - Junction Capacitance

\[ V_D > 0 \]

- PN Junction Under Forward Bias
  - I/V Characteristics
Carrier Diffusion across the Junction

- Because of the difference in hole and electron concentrations on each side of the junction, carriers diffuse across the junction:

Notation:
- $n_n \equiv$ electron concentration on N-type side (cm$^{-3}$)
- $p_n \equiv$ hole concentration on N-type side (cm$^{-3}$)
- $p_p \equiv$ hole concentration on P-type side (cm$^{-3}$)
- $n_p \equiv$ electron concentration on P-type side (cm$^{-3}$)
Depletion Region

- As conduction electrons and holes diffuse across the junction, they leave behind ionized dopants. Thus, a region that is depleted of mobile carriers is formed.
  - The charge density in the depletion region is not zero.
  - The carriers which diffuse across the junction recombine with majority carriers, i.e. they are annihilated.
Carrier Drift across the Junction

• Because charge density ≠ 0 in the depletion region, an electric field exists, hence there is drift current.

Gauss' Law: \( (I-D) \)
\[
\frac{dE}{dx} = \frac{P}{\varepsilon_i}
\]

balanced charge:
\[
bN_D = aN_A
\]

\( \Rightarrow \) depletion width is larger on the more lightly doped side.

\( E = -\frac{dV}{dx} \)

area = voltage dropped across depletion region

EE105 Fall 2007
Lecture 2, Slide 19
Prof. Liu, UC Berkeley
PN Junction in Equilibrium

- In equilibrium, the drift and diffusion components of current are balanced; therefore the net current flowing across the junction is zero.

\[
J_{p,\text{drift}} = -J_{p,\text{diff}} \\
J_{n,\text{drift}} = -J_{n,\text{diff}} \\
J_{\text{tot}} = J_{p,\text{drift}} + J_{n,\text{drift}} + J_{p,\text{diff}} + J_{n,\text{diff}} = 0
\]
Built-in Potential, $V_0$

- Because of the electric field in the depletion region, there exists a potential drop across the junction:

$$q\mu_p E = qD_p \frac{dp}{dx} \Rightarrow p \mu_p \left( -\frac{dV}{dx} \right) = D_p \frac{dp}{dx}$$

$$\Rightarrow -\mu_p \int_{x_1}^{x_2} dV = D_p \int_{p_n}^{p_p} \frac{dp}{p}$$

$$\Rightarrow V(x_1) - V(x_2) = \frac{D_p}{\mu_p} \ln \frac{p_p}{p_n} = \frac{kT}{q} \ln \left( \frac{N_A}{n_i^2 / N_D} \right)$$

$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

(Unit: Volts)
• Estimate the built-in potential for PN junction below.

- Note that $\frac{kT}{q} \ln(10) \approx 26\text{mV} \times 2.3 \approx 60\text{mV}$

$$V_0 = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = \frac{kT}{q} \ln\left(\frac{10^{18} \cdot 10^{15}}{10^{26}}\right) = \frac{kT}{q} \ln(10^{13}) = 13.60\text{ mV} \approx 0.78\text{ V}$$

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<td>$N_D = 10^{18} \text{ cm}^{-3}$</td>
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PN Junction under Reverse Bias

- A reverse bias increases the potential drop across the junction. As a result, the magnitude of the electric field increases and the width of the depletion region widens.

\[ W_{dep} = \sqrt{\frac{2\varepsilon_{si}}{q}} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R) \]

\[ W_{dep} = \sqrt{\frac{2\varepsilon_{si}}{qN}} (V_0 + V_R) \]

\( N = \text{net doping on lightly doped side} \)

Typically \( 10^{-6} \mu m < W_{dep} < 1 \mu m \)
Diode Current under Reverse Bias

- In equilibrium, the built-in potential effectively prevents carriers from diffusing across the junction.
- Under reverse bias, the potential drop across the junction increases; therefore, negligible diffusion current flows. A very small drift current flows, limited by the rate at which minority carriers diffuse from the quasi-neutral regions into the depletion region.
A reverse-biased PN junction can be viewed as a capacitor. The depletion width \( W_{dep} \) and hence the junction capacitance \( C_j \) varies with \( V_R \).

\[
C_j = \frac{\varepsilon_{si}}{W_{dep}}
\]

Units: \( \text{F/cm}^2 \)

\( C \Delta V_D = \Delta Q \)
Voltage-Dependent Capacitance

\[ C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} \]

\[ C_{j0} = \sqrt{\frac{\varepsilon_{si} q}{2}} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_0} \]

\( \varepsilon_{si} \approx 10^{-12} \text{ F/cm} \) is the permittivity of silicon.
Reverse-Biased Diode Application

- A very important application of a reverse-biased PN junction is in a voltage controlled oscillator (VCO), which uses an LC tank. By changing $V_R$, we can change $C$, which changes the oscillation frequency.

$$f_{res} = \frac{1}{2\pi \sqrt{LC}}$$
Summary

- Current flowing in a semiconductor is comprised of drift and diffusion components: 
  \[ J_{\text{tot}} = q p \mu_p E + q n \mu_n E + q D_n \frac{dn}{dx} - q D_p \frac{dp}{dx} \]
- A region depleted of mobile charge exists at the junction between P-type and N-type materials.
  - A built-in potential drop \((V_0)\) across this region is established by the charge density profile; it opposes diffusion of carriers across the junction. A reverse bias voltage serves to enhance the potential drop across the depletion region, resulting in very little (drift) current flowing across the junction.
  - The width of the depletion region \((W_{\text{dep}})\) is a function of the bias voltage \((V_D)\).
    \[
    W_{\text{dep}} = \sqrt{\frac{2 \varepsilon_{si}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \left( V_0 - V_D \right)}
    \]
    \[
    V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}
    \]