CS 283
Advanced Computer Graphics

Mesh Simplification

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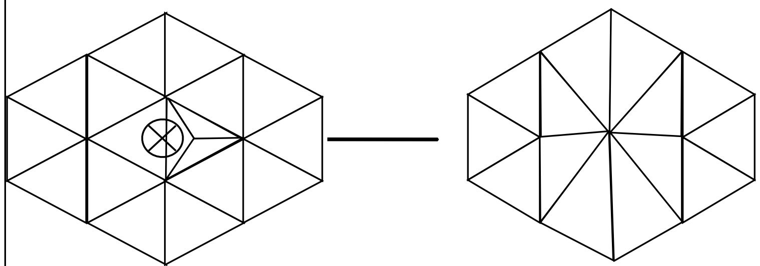
Based on slides by Ravi Ramamoorthi

Appearance Preservation

Caltech & Stanford Graphics Labs and Jonathan Cohen

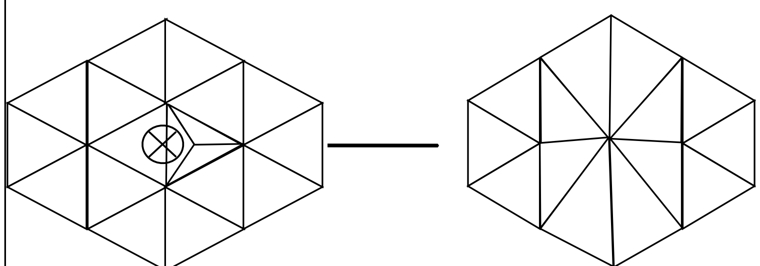
Mesh Decimation

- Basic simplification operation is edge collapse

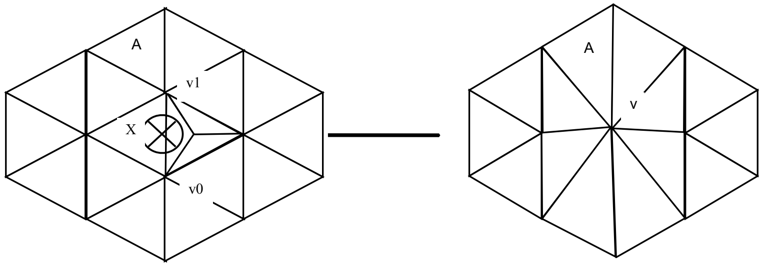


Mesh Decimation

- Basic simplification operation is edge collapse

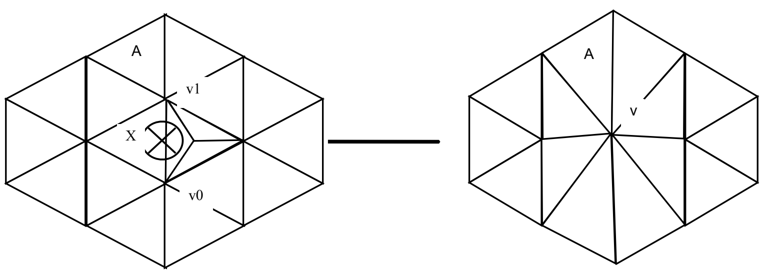


Edge Collapse



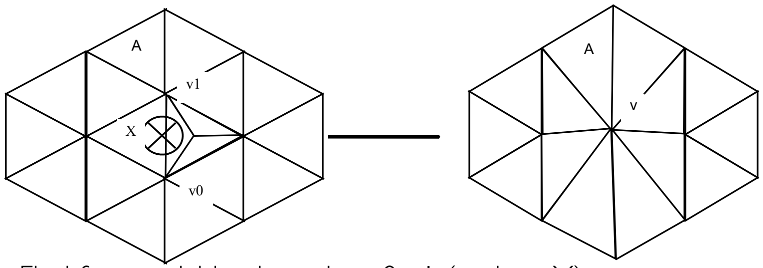
- Create new vertex v (based on appropriate rule)
- Find all faces/edges neighbor vertex v1 (such as A)
- Change them to use v instead of v1. Do the same for v0
- Depend on data structure, you need to fix all faces, edges

Edge Collapse



- Create new vertex v
  - based on appropriate rule like average
- Find all faces that neighbor vertex v1 (such as A)
  - Simple use of vertex to face adjacency
- Change them to use v instead of v1. Do the same for v0

Edge Collapse



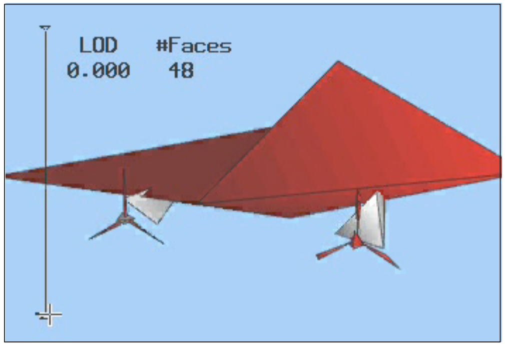
- Find faces neighboring edge v0-v1 (such as X)
- Remove from mesh
  - This may involve updating face/vertex adjacency relationships etc.
  - e.g. what is adjacency for v (faces adjacent to vertex?)
  - Are other vertices affected in terms of adjacent faces?
- Worry about triangle fins

Progressive Meshes

- Write edge collapses to file
- Read in file and invert order
- Key idea is vertex-split (opposite of edge-collapse)
- Include some control to make model coarser/finer

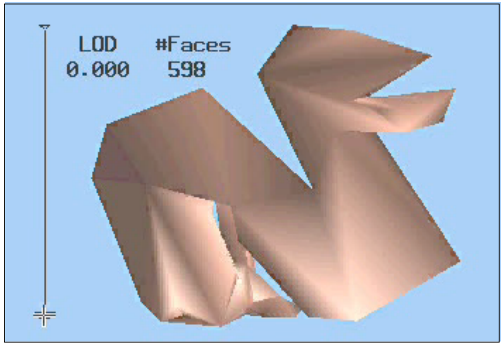
- e.g. Hoppe geomorph demo

Progressive Meshes



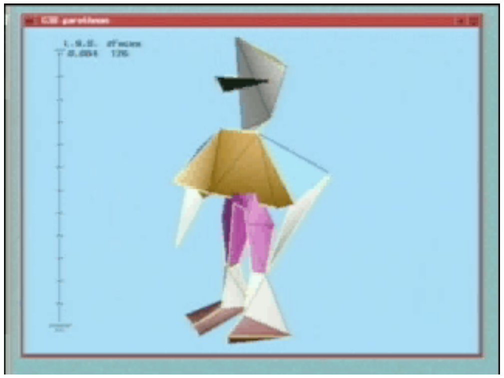
Hoppe

Progressive Meshes



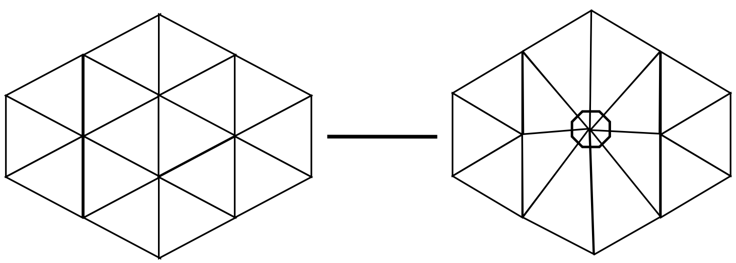
Hoppe

Geomorph Demo



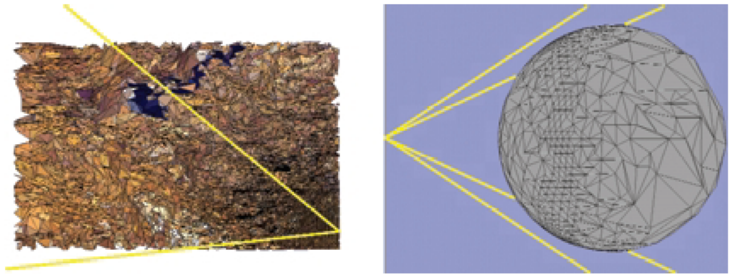
Hoppe

Reverse of Edge Collapse



View-Dependent Simplification

- Simplify dynamically according to viewpoint
  - Visibility
  - Silhouettes
  - Lighting



Hoppe

Quadric Error Metrics

- Garland & Heckbert, SIGGRAPH 97
  - Greedy decimation algorithm
- Pair collapse (allow edge + non-edge collapses)
- Quadric error metrics:
  - Evaluate potential collapses
  - Determine optimal new vertex locations

Background: Computing Planes

- Each triangle in mesh has associated plane

$$ax + by + cz + d = 0$$

- For a triangle, find its (normalized) normal using cross products

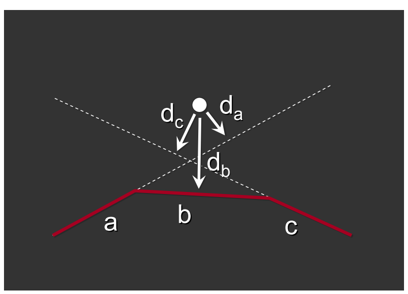
$$\vec{n} = \frac{AB \times AC}{|AB \times AC|} \qquad \vec{n} \bullet \vec{v} - \vec{A} \bullet \vec{n} = 0$$

- Plane equation?

$$\vec{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad d = -\vec{A} \bullet \vec{n}$$

Quadric Error Metrics

- Based on point-to-plane distance
- Better quality than point-to-point



Quadric Error Metrics

- Quadric Error Metrics

$$\Delta_{\mathbf{v}} = \sum_{\mathbf{p}} Dist(\mathbf{v},\mathbf{p})^2$$

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$Dist(\mathbf{v},\mathbf{p}) = ax + by + cz + d = \mathbf{p}^T\mathbf{v}$$

Quadric Error Metrics

$$\begin{aligned} \Delta &= \sum_{\mathbf{p}} (\mathbf{p}^T\mathbf{v})^2 \\ &= \sum_{\mathbf{p}} \mathbf{v}^T\mathbf{p}\mathbf{p}^T\mathbf{v} \\ &= \mathbf{v}^T\left(\sum_{\mathbf{p}} \mathbf{p}\mathbf{p}^T\right)\mathbf{v} \\ &= \mathbf{v}^T\mathbf{Q}\mathbf{v} \end{aligned}$$

- Common mathematical trick: quadratic form = symmetric matrix Q multiplied twice by a vector
- Initially, distance to all planes 0, net is 0 for all vertices

Using Quadrics

- Approximate error of edge collapses
- Each vertex v has associated quadric Q
- Error of collapsing v1 and v2 to v' is $v'^T Q_1 v' + v'^T Q_2 v'$
- Quadric for new vertex v' is Q'=Q1+Q2

Using Quadrics

- Find optimal location v' after collapse:

$$\mathbf{Q'} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ q_{14} & q_{24} & q_{34} & q_{44} \end{bmatrix}$$

$$\min_{\mathbf{v'}} \mathbf{v'}^T \mathbf{Q'} \mathbf{v'} : \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

Using Quadrics

- Find optimal location v' after collapse:

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{v'} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

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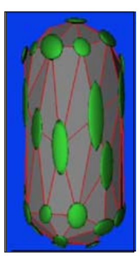
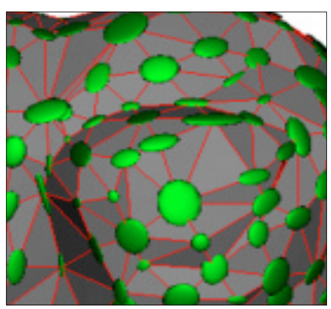
Quadric Visualization

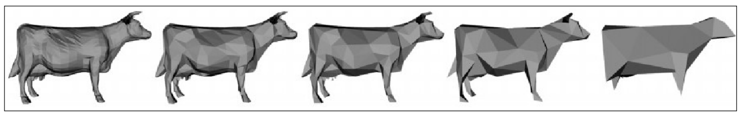
- Ellipsoids: iso-error surfaces
- Smaller ellipsoid = greater error for a given motion
- Lower error for motion parallel to surface
- Lower error in flat regions than at corners
- Elongated in "cylindrical" regions





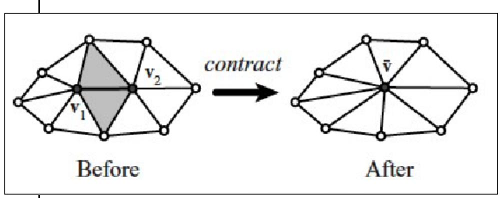
Surface Simplification

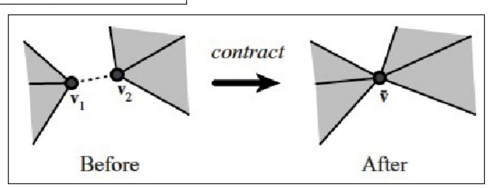
- Efficiency (70000 to 100 faces in 15s in 1997)
- High quality, feature preserving (primary appearance emphasized rather than topology)
- Generality, non-manifold models, collapse disjoint regions



Simplification

- Pair contractions in addition to edge collapses
- Previously connected regions may come together





Algorithm Outline

- Restrict process to a set of *valid pairs*:
  - $(\mathbf{v}_i, \mathbf{v}_j)$ is an edge, or
  - $||\mathbf{v}_i - \mathbf{v}_j|| < t$, a threshold
    - $t = 0$ restricts to edge contraction
    - $t \gg 0$ can connect distant regions or yield $O(n^2)$ pairs
- Iteratively remove *best* pair and update valid pairs list:
  - Each vertex has a set with the pairs it belongs to:
    - $\mathbf{v}_i \rightarrow \text{Pairs}(\mathbf{v}_i)$
    - $(\mathbf{v}_i, \mathbf{v}_j) \rightarrow \overline{\mathbf{v}} \Rightarrow \text{Pairs}(\overline{\mathbf{v}}) = \text{Pairs}(\mathbf{v}_i) \cup \text{Pairs}(\mathbf{v}_j)$
- But how to choose *best* pair?

Use quadric error

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Final Algorithm

- Compute Q_i for all vertices $\mathbf{v}_i$
- Determine valid pairs
- Compute optimal contraction target and associated quadric error for each pair
- Place all pairs in a heap, ordered by smallest error
- Repeat
  - Get least error pair $(\mathbf{v}_i, \mathbf{v}_j)$ from heap
  - Contract pair (move edges to $\overline{\mathbf{v}}$, remove degenerate planes)
  - Update cost for all pairs involving $\mathbf{v}_i$ and $\mathbf{v}_j$
- Until done.

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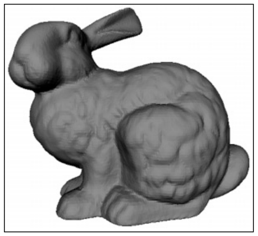
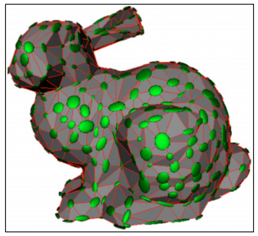
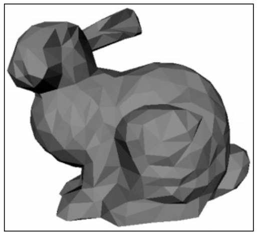
Results



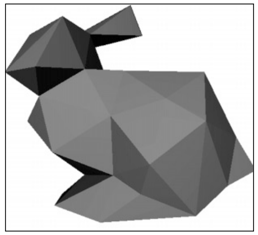
Original



Quadrics

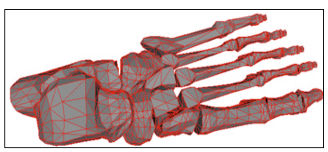


1k tris

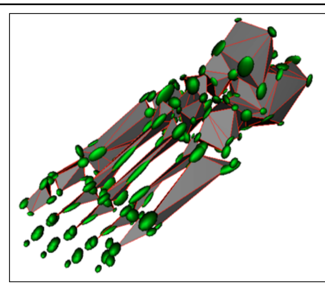


100 tris

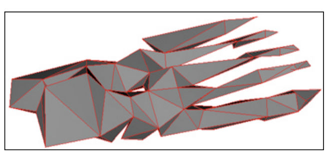
Results



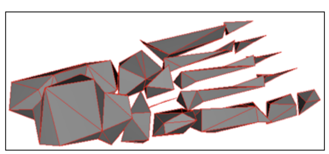
Original



Quadrics



250 tris



250 tris, edge collapses only

Additional Details

- Preserving boundaries/discontinuities (weight quadrics by appropriate penalty factors)
- Preventing mesh inversion (flipping of orientation): compare normal of neighboring faces, before after
- Has been modified for many other applications
  - e.g. in silhouettes, want to make sure volume always increases, never decreases
  - Take color and texture into account (followup paper)
- See paper, other more recent works for details