Foundations of Computer Graphics (Spring 2010)
CS 184, Lecture 20: Global Illumination
http://inst.eecs.berkeley.edu/~cs184

Illumination Models
So far considered mainly local illumination
- Light directly from light sources to surface
Global Illumination: multiple bounces
- Already ray tracing: reflections/refractions

Some images courtesy Henrik Wann Jensen

Global Illumination
Diffuse interreflection, color bleeding [Cornell Box]

Global Illumination
Caustics: Focusing through specular surface
Major research effort in 90s, 90s till today

Overview of lecture
- Theory for all methods (ray trace, radiosity)
- We derive Rendering Equation [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases

Fairly theoretical lecture (but important). Not well covered in any of the textbooks. Closest are 2.6.2 in Cohen and Wallace handout (but uses slightly different notation, argument [swaps x, x’ among other things])

Outline
- Reflectance Equation (review)
- Global Illumination
  - Rendering Equation
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)
Reflectance Equation (review)

\[ L_i(x, \omega_i) = L_e(x, \omega_i) + \int \frac{L_e(x, \omega_i) f(x, \omega_i, \omega_r) \cos \theta}{dA} \]

- \( L_i \): Reflected Light (Output Image)
- \( L_e \): Emission Incident Light (from light source)
- \( f \): BRDF
- \( \omega_i \), \( \omega_r \): Cosine of Incident angle

Replace sum with integral

Global Illumination

\[ L_i(x, \omega_i) = L_i(x, \omega_i) + \int_{\Omega} L_e(x, \omega_i) f(x, \omega_i, \omega_r) \cos \theta \, d\omega_r \]

- \( L_i \): Reflected Light (Output Image)
- \( L_e \): Reflected Light (from surface)
- \( f \): BRDF
- \( \omega_i \), \( \omega_r \): Cosine of Incident angle

Rendering Equation (Kajiya 86)

\[ L_i(x, \omega_i) = L_e(x, \omega_i) + \int_{\Omega} L_e(x', \omega_i) f(x, \omega_i, \omega_r) \cos \theta \, d\omega_r \]

- \( L_i \): Reflected Light (Output Image)
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Surfaces (interreflection)

\[ L_i(x, \omega_i) = L_i(x, \omega_i) + \int_{\Omega} L_e(x', \omega_i) f(x, \omega_i, \omega_r) \cos \theta \, d\omega_r \]

- \( L_i \): Reflected Light (Output Image)
- \( L_e \): Reflected Light (from surface)
- \( f \): BRDF
- \( \omega_i \), \( \omega_r \): Cosine of Incident angle

Figure 4: A sample image. All objects are colored gray. Colors on the object are due to reradiation from the green glass ball and other bleeding from the browser geometry.
Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- **As a general Integral Equation and Operator**
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)

The material in this part of the lecture is fairly advanced and not covered in any of the texts. The slides should be fairly complete. This section is fairly short, and I hope some of you will get some insight into solutions for general global illumination.

Rendering Equation as Integral Equation

\[
I(u) = e(u) + \int I(v) K(u, v) dv
\]

Kernel of equation

Light Transport Operator

\[ L = E + KL \]

Can be discretized to a simple matrix equation [or system of simultaneous linear equations] (L, E are vectors, K is the light transport matrix)

Solution Techniques

All global illumination methods try to solve (approximations of) the rendering equation

- Too hard for analytic solution: numerical methods
- General theory of solving integral equations

Radiosity (next lecture; usually diffuse surfaces)

- General class numerical finite element methods (divide surfaces in scene into a finite set elements or patches)
- Set up linear system (matrix) of simultaneous equations
- Solve iteratively

Ray Tracing and extensions

- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

\[ L = E + KL \]

\[ IL - KL = E \]

\[ (I - K)L = E \]

\[ L = (I - K)^{-1} E \]

Binomial Theorem

\[ L = (I + K + K^2 + K^3 + \ldots) E \]

\[ L = E + KE + K^2 E + K^3 E + \ldots \]

Ray Tracing

\[ L = E + KE + K^2 E + K^3 E + \ldots \]

Emission directly

From light sources

Direct Illumination on surfaces

Global Illumination

(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect)
[Caustics etc]
Ray Tracing

\[ L = E + KE + K^2E + K^3E + \ldots \]

- Emission directly from light sources
- Direct illumination on surfaces
- Global illumination (One bounce indirect)
  - [Mirrors, Refraction]
- Global illumination (Two bounce indirect)
  - [Caustics etc]

OpenGL Shading

- Reflectance Equation (review)
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Overview

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- We derive *Rendering Equation* [Kajiya 86]
  - Major theoretical development in field
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