CS-184: Computer Graphics

Lecture #22: Spring and Mass systems

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Today

- Spring and Mass systems
  - Distance springs
  - Spring dampers
  - Edge springs

Hanging Cloth

Huamin Wang, Ravi Ramamoorthi, and James F. O'Brien. "**Data-Driven Elastic Models for Cloth: Modeling and Measurement**". *ACM Transactions on Graphics*, 30(4):71:1–11, July 2011. Proceedings of ACM SIGGRAPH 2011, Vancouver, BC Canada.

Walking Mannequin

Huamin Wang, Ravi Ramamoorthi, and James F. O'Brien. "**Data-Driven Elastic Models for Cloth: Modeling and Measurement**". *ACM Transactions on Graphics*, 30(4):71:1–11, July 2011. Proceedings of ACM SIGGRAPH 2011, Vancouver, BC Canada.

A Simple Spring

- Ideal ***zero***-length spring

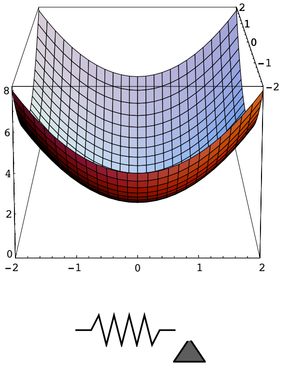
$$\boldsymbol{f}_{a\rightarrow b} = k_s(\boldsymbol{b} - \boldsymbol{a})$$

$$\boldsymbol{f}_{b\rightarrow a} = -\boldsymbol{f}_{a\rightarrow b}$$

- Force pulls points together
- Strength proportional to distance

A Simple Spring

- Energy potential



$$E = 1/2\ k_s(\boldsymbol{b} - \boldsymbol{a}) \cdot (\boldsymbol{b} - \boldsymbol{a})$$

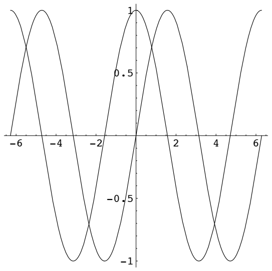
$$\boldsymbol{f}_{a\rightarrow b} = k_s(\boldsymbol{b} - \boldsymbol{a})$$

$$\boldsymbol{f}_{b\rightarrow a} = -\boldsymbol{f}_{a\rightarrow b}$$

$$\boldsymbol{f}_a = -\nabla_a E = -\left[\frac{\partial E}{\partial a_x}, \frac{\partial E}{\partial a_y}, \frac{\partial E}{\partial a_z}\right]$$

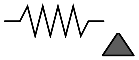
A Simple Spring

- Energy potential: kinetic **vs** elastic



$$E = 1/2\ k_S(\boldsymbol{b} - \boldsymbol{a}) \cdot (\boldsymbol{b} - \boldsymbol{a})$$

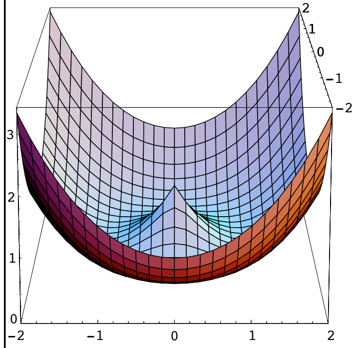
$$E = 1/2\ m(\dot{\boldsymbol{b}} - \dot{\boldsymbol{a}}) \cdot (\dot{\boldsymbol{b}} - \dot{\boldsymbol{a}})$$



Non-Zero Length Springs

$$\boldsymbol{f}_{a \rightarrow b} = k_S \frac{\boldsymbol{b} - \boldsymbol{a}}{||\boldsymbol{b} - \boldsymbol{a}||} (||\boldsymbol{b} - \boldsymbol{a}|| - l)$$

Rest length



$$E = k_S\,(||\boldsymbol{b} - \boldsymbol{a}|| - l)^2$$

Comments on Springs

- Springs with zero rest length are linear
- Springs with non-zero rest length are nonliner
  - Force ***magnitude*** linear w/ discplacement (from rest length)
  - Force direction is non-linear
  - Singularity at

$$||\boldsymbol{b} - \boldsymbol{a}|| = 0$$

Damping

- “Mass proportional” damping

$$\xleftarrow{f} \quad \xrightarrow{\dot{a}} \qquad \boldsymbol{f} = -k_d \dot{\boldsymbol{a}}$$

  - Behaves like viscous drag on all motion
  - Consider a pair of masses connected by a spring
    - How to model rusty ***vs*** oiled spring
    - Should internal damping slow group motion of the pair?
  - Can help stability... up to a point

Damping

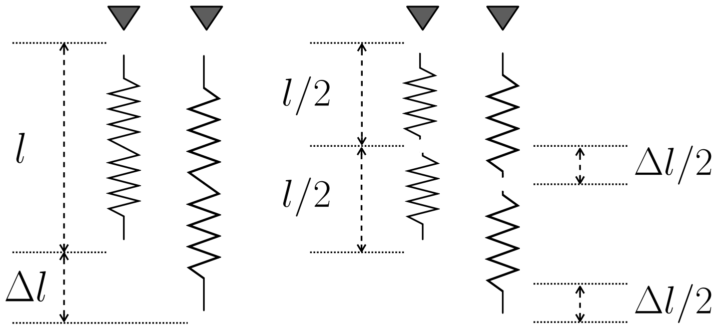
- "Stiffness proportional" damping

$$\boldsymbol{f}_a = -k_d \frac{\boldsymbol{b}-\boldsymbol{a}}{||\boldsymbol{b}-\boldsymbol{a}||^2}(\boldsymbol{b}-\boldsymbol{a})\cdot(\dot{\boldsymbol{b}}-\dot{\boldsymbol{a}})$$

  - Behaves viscous drag on change in spring length
  - Consider a pair of masses connected by a spring
    - How to model rusty ***vs*** oiled spring
    - Should internal damping slow group motion of the pair?

Spring Constants

- Two ways to model a single spring



Spring Constants

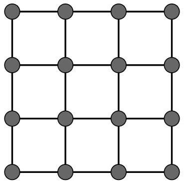
- Constant k_S gives inconsistent results with different discretizations
- Change in length is not what we want to measure
- Strain: change in length as fraction of original length

$$\epsilon = \frac{\Delta l}{l_0}$$

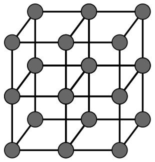
Nice and simple for 1D...

Structures from Springs

- Sheets



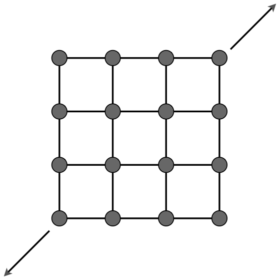
- Blocks



- Others

Structures from Springs

- They behave like what they are (obviously!)

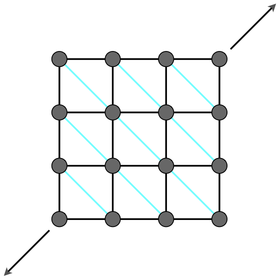


This structure will not resist shearing

This structure will not resist out-of-plane bending either...

Structures from Springs

- They behave like what they are (obviously!)

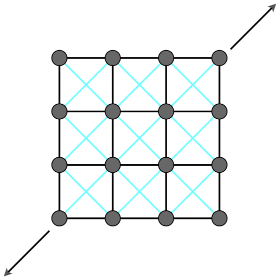


This structure will resist shearing but has anisotopic bias

This structure still will not resist out-of-plane bending

Structures from Springs

- They behave like what they are (obviously!)

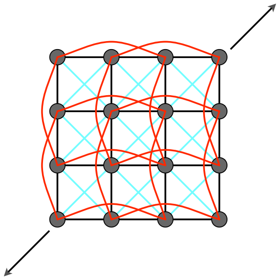


This structure will resist shearing
Less bias
Interference between spring sets

This structure still will not resist out-of-plane bending

Structures from Springs

- They behave like what they are (obviously!)

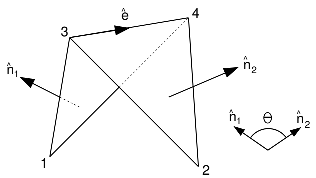


This structure will resist shearing
Less bias
Interference between spring sets

This structure will resist out-of-plane bending
Interference between spring sets
Odd behavior

How do we set spring constants?

Edge Springs



$$u_1 = |E| \frac{N_1}{|N_1|^2} \qquad u_2 = |E| \frac{N_2}{|N_2|^2}$$

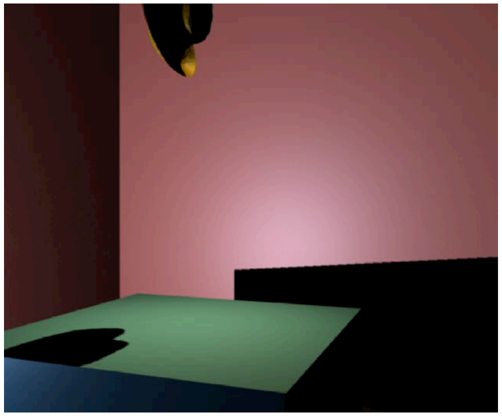
$$u_3 = \frac{(x_1 - x_4) \cdot E}{|E|} \frac{N_1}{|N_1|^2} + \frac{(x_2 - x_4) \cdot E}{|E|} \frac{N_2}{|N_2|^2}$$

$$u_4 = -\frac{(x_1 - x_3) \cdot E}{|E|} \frac{N_1}{|N_1|^2} - \frac{(x_2 - x_3) \cdot E}{|E|} \frac{N_2}{|N_2|^2}$$

$$F_i^e = k^e \frac{|E|^2}{|N_1| + |N_2|} \sin(\theta/2)\, u_i$$

From Bridson ***et al.,*** 2003, also see Grinspun ***et al.,*** 2003

Example: Thin Material



Discrete Shells
SCA 2003
Eitan Grinspun, Anil Hirani, Mathieu Desbrun and Peter Schröder

Strain Limiting

Bunny

Hollow Triangle Mesh

59K Elements

Huamin Wang, James F. O'Brien, and Ravi Ramamoorthi. "**Multi-Resolution Isotropic Strain Limiting**". In *Proceedings of ACM SIGGRAPH Asia 2010*, pages 160:1–10, December 2010.

Suggested Reading

- Physically Based Modeling: Principles and Practice
  - Andy Witkin and David Baraff
  - http://www-2.cs.cmu.edu/~baraff/sigcourse/index.html
- Grinspun, Hirani, Desbrun, and Peter Schroder, "Discrete Shells," SCA 2003
- Bridson, Marino, and Fedkiw, "Simulation of Clothing with Folds and Wrinkles," SCA 2003
- O'Brien and Hodgins, "Graphical Modeling and Animation of Brittle Fracture," SIGGRAPH 99