CS-184: Computer Graphics

Lecture #16: Global Illumination

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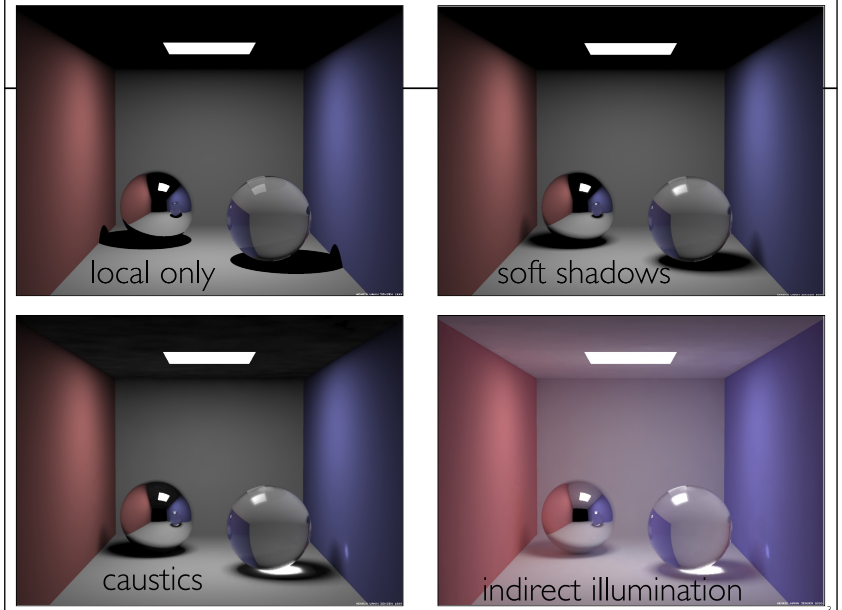
University of California, Berkeley

V2009-F-16-1.0

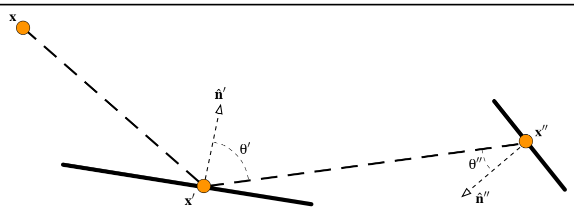
Today

- The Rendering Equation
- Radiosity Method
- Photon Mapping
- Ambient Occlusion

Sunday, November 8, 2009

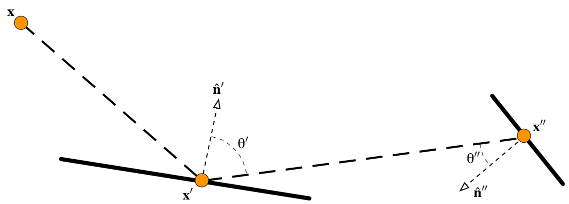


The Rendering Equation



The light shining on x from x' is equal to:
- the emitted light from x' toward x, plus
- for each bit of surface in the scene, how much light shines from that bit onto x' and is reflected toward x, scaled appropriately

The Rendering Equation



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$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}') \left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'') L_s(\mathbf{x}',\mathbf{x}'') \frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2} \mathrm{d}\mathbf{x}'' \right]$$

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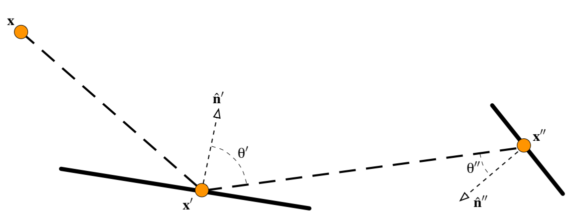

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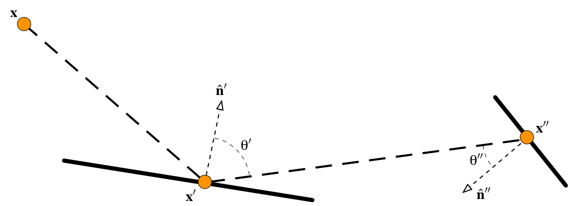
Light energy hitting x from x'



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The Rendering Equation

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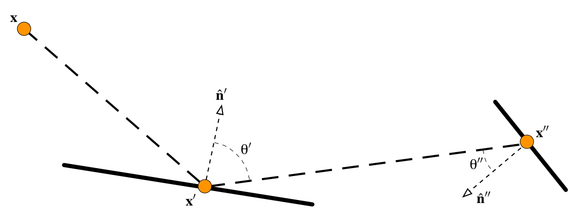

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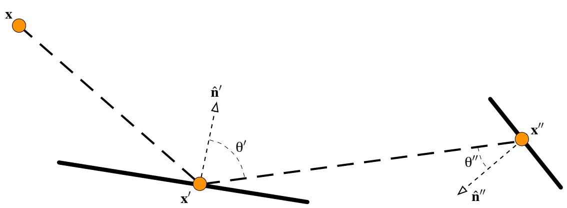
Can x see x' ?



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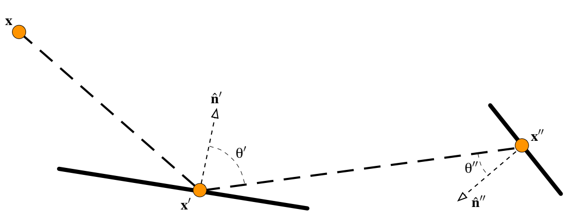

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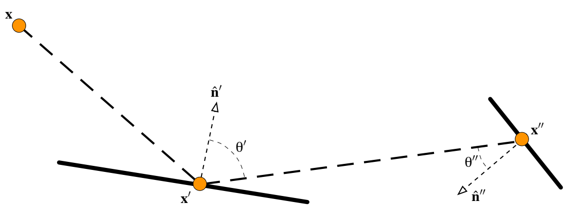
Light emitted from x' toward x



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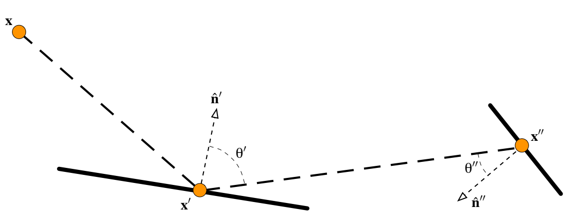

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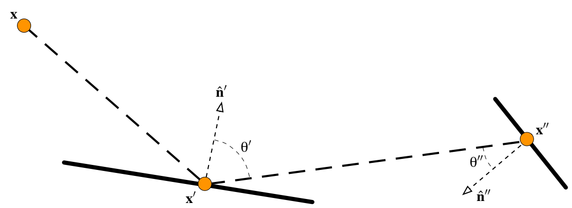
sum over every bit of surface in the scene



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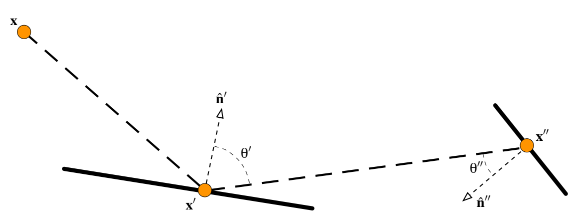

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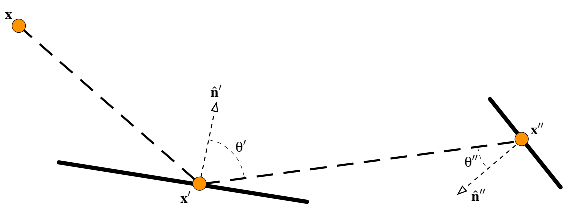
Light emitted from x'' toward x'



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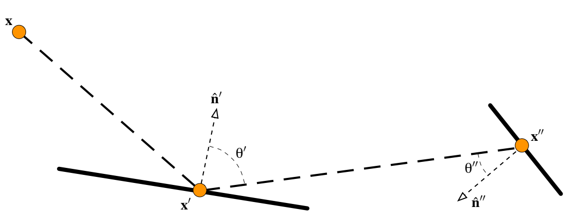

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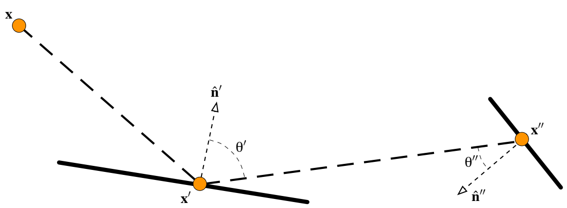
scaled down by the BRDF of x'



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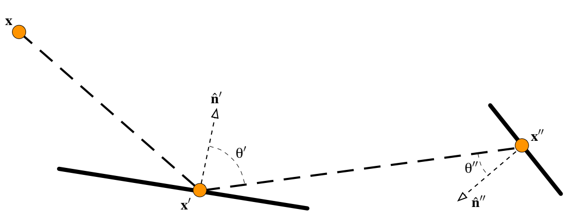

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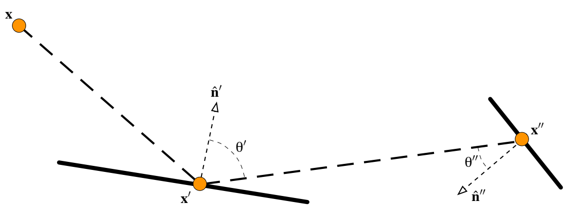
scaled down by distance and relative orientation ("form factor")



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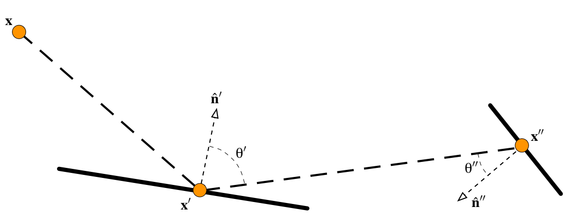


Radiosity

- Assume all materials are perfectly Lambertian (diffuse only, no specularities)
  - Removes all dependance on directions
  - Reduces dimensionality of lightfield
  - Allows a FEM solution (break up into chunks)
- Can also relax assumption slightly...

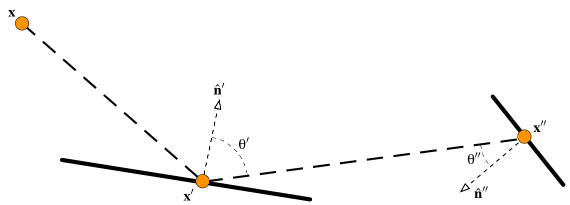
Early radiosity

from Hanrahan 2000

Assume Lambertian

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'')L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''\right]$$

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E_{x'} + \int_S \rho_{x'}L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''\right]$$



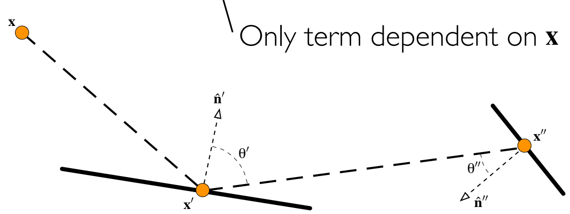
8

Assume Lambertian

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'')L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''\right]$$

$$L_s(\mathbf{x},\mathbf{x}') = \underline{\delta(\mathbf{x},\mathbf{x}')}\left[E_{x'} + \int_S \rho_{x'}L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''\right]$$

Only term dependent on $\mathbf{x}$



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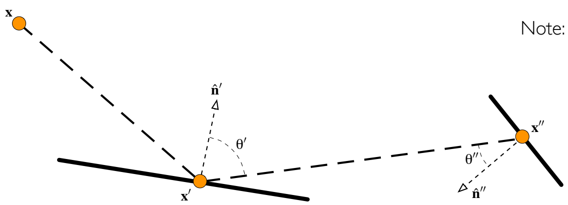
Rewrite in Terms of Radiosity

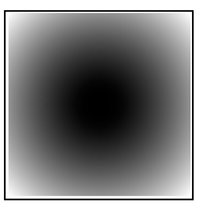
$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E_{x'} + \int_S \rho_{x'} L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''\right]$$

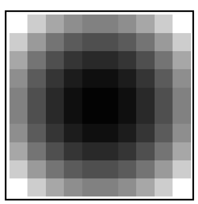
$$H_{x'} = E_{x'} + \rho_{x'}\int_S \delta(\mathbf{x}',\mathbf{x}'')\frac{H_{x''}}{2\pi}\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''$$



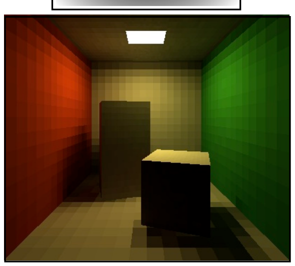
Note: we changed defn of E here.

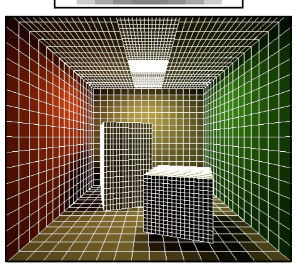
Discretize into Patches





Piece-wise constant patches





Example mesh for Cornell Box
by Mark Schmelzenbach

Discretize into Patches

The Candlestick Theater,
Mark Mack Architects.

Discretize into Patches

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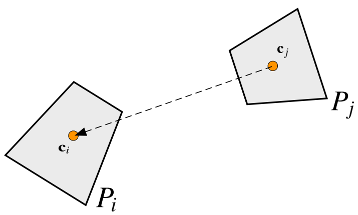
Rewrite in Terms of Patches

$$H_{x'} = E_{x'} + \rho_{x'} \int_S \delta(\mathbf{x}', \mathbf{x}'') \frac{H_{x''}}{2\pi} \frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}' - \mathbf{x}''||^2} \mathrm{d}\mathbf{x}''$$

$$H_i = E_i + \rho_i \sum_j H_j \int_{S_j} \delta_{ij} \frac{\cos(\theta_i)\cos(\theta_j)}{2\pi||\mathbf{c}_i - \mathbf{x}||^2} \mathrm{d}\mathbf{x}$$



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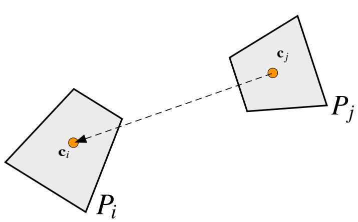
Form factor from **j** to **i**, F_{ij}



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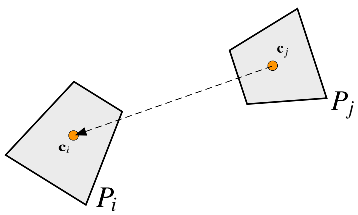
Form factor from ***j*** to ***i***, F_{ij}



Example of a rough approximation:

$$F_{ij} \approx \delta_{ij} \frac{\cos(\theta_i)\cos(\theta_j)}{2\pi||\mathbf{c}_i - \mathbf{c}_j||^2} A_j$$

Radiosity Method

- Given the E_i and ρ_i
- First compute F_{ij}
- Then solve $H_i = E_i + \rho_i \sum_j H_j F_{ij}$

$$\mathbf{h} = \mathbf{e} + \mathbf{A}\mathbf{h}$$
$$(\mathbf{I} - \mathbf{A})\mathbf{h} = \mathbf{e}$$

- Comments:
  - The matrix $\mathbf{A}$ is typically very large
  - It is also sparse (why?)
  - Should be solved with an iterative method
    - *e.g.:* Jacobi or Gauss-Seidel
  - **Solution is view independent**

Radiosity Method

- Given the light emitted and surface properties
- First compute F_{ij} , form factors between patches
- Then **solve a linear system to balance energy between all patches**
- Comments:
  - The system is very large
  - It is also sparse (why?)
  - Should be solved with an iterative method
    - e.g.: Jacobi or Gauss-Seidel
  - **Solution is view independent**

Progressive Radiosity

- If magnitude of eigenvalues of $\mathbf{A}<1$

$$(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\cdots$$

  - True for form-factor matrices

  - Use Gauss-Seidel-like iteration but reorder by priority

$$\mathbf{h}^{k+1} = \mathbf{h}^k + \mathbf{u}^{k+1}$$

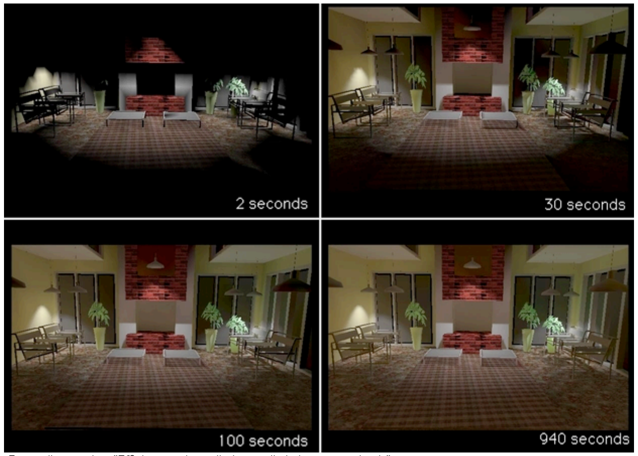
$$\mathbf{u}^{k+1} = \mathbf{A}\,\mathbf{u}^k$$

$$\mathbf{h}^0 = 0 \quad \mathbf{u}^0 = \mathbf{e}$$

Idea: let important sources of light energy emit first, maybe don't even bother with dark things

Southwell Relaxation

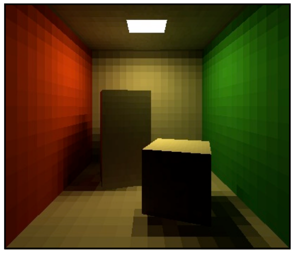
Progressive Radiosity



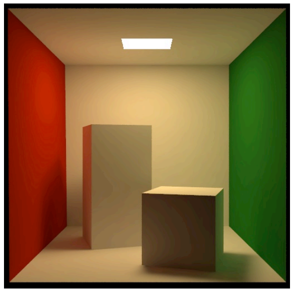
From dissertation "Efficient and predictive realistic image synthesis"
by Karol Myszkowski

Touchup

- Each patch will have a constant color
  - Smooth solution (**e.g.** average to vertices)



Example mesh for Cornell Box
by Mark Schmelzenbach



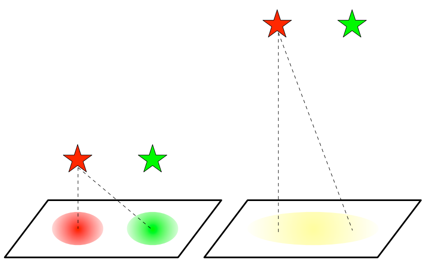
Does not match but you get the idea...

Other Things

- Each patch will have a constant color
  - Smooth solution (**e.g.** average to vertices)
- No specular reflection
  - Add Phong specular term or raytraced specular reflection
- Grid artifacts
  - Be clever with grid...

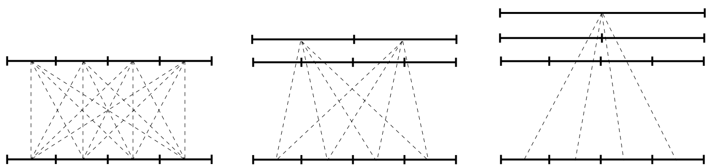
Hierarchical Radiosity

- Light smoothes with distance
  - Compare $1/h^2$ with $1/(h^2+d^2)$ as h gets large



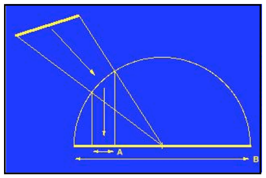
Hierarchical Radiosity

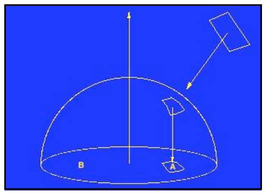
- Light smoothes with distance
  - Compare $1/h^2$ with $1/(h^2+d^2)$ as h gets large
- Group patches into hierarchy
  - Far interactions use lower-res form factors



Computing Form Factors

- Form factors have a geometric meaning

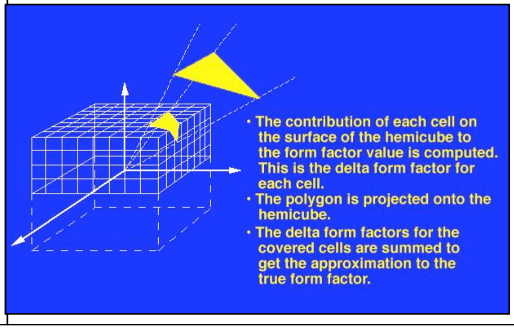




Images from
SIGGRAPH 93 Education Slide Set
by Stephen Spencer

Computing Form Factors

- Form factors have a geometric meaning
- "Hemicube" algorithm uses regular scan conversion



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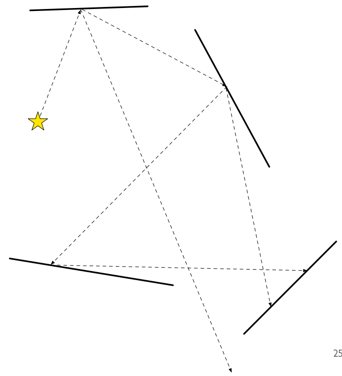
Computing Form Factors

- Form factors have a geometric meaning
- "Hemicube" algorithm uses regular scan conversion
- Also computed by ray-based sampling
- **In practice, computing form factors is the bottleneck**

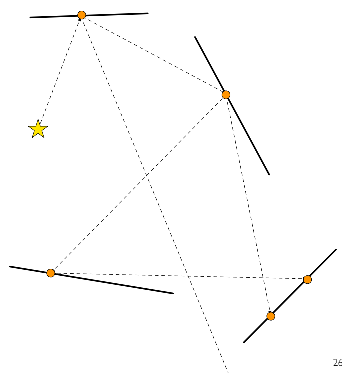
Photon Mapping

- Lights cast "photons" into environment
  - Cast in random directions
  - Trace into environment
  - Store records at intersections

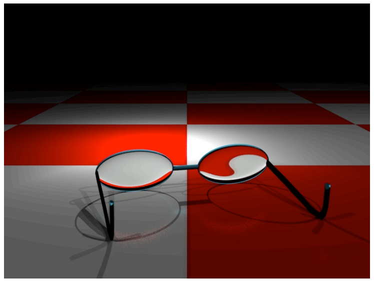


Photon Mapping

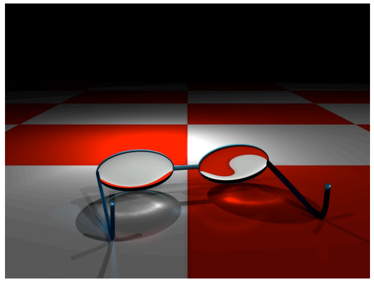
- Lights cast "photons" into environment
  - Cast in random directions
  - Trace into environment
  - Store records at intersections
    - With KD-Trees...



Comparison



Ray Tracing



Ray Tracing w/ Photon Map

Catherine Bendebury and Jonathan Michaels
CS 184 Spring 2005

Photon Mapping

Image by Per Christensen

A ray traced image

Note:
- Dark shadows
- Unlit corners
- Nice reflections

Photon Mapping

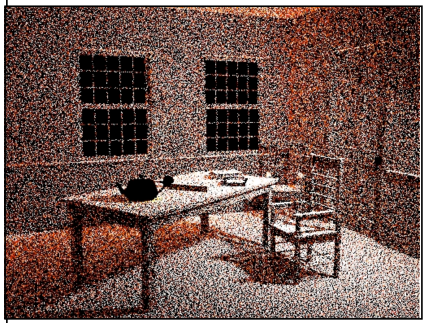


Image by Per Christensen

Raw photons

Note:
- Noisy
- Sparse

Photon Mapping

Image by Per Christensen

Interpolated Photons

Note:
- Still noisy
- Biased

Photon Mapping

Image by Per Christensen

Interpolated Photons (multiplied by diffuse)

Note:
Still noisy
Biased

Photon Mapping

- Final Gather
  - Ray trace scene
  - Direct and specular rays as normal
  - Diffuse rays traced into photon map

  - *Diffuse reflection smoothes noise*

Photon Mapping

Image by Per Christensen

Final Image

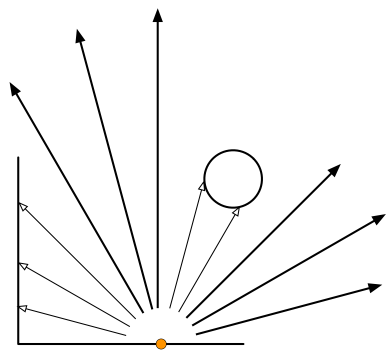
Note:
- Not noisy
- Nice lighting
- Reflections
- May still be biased

Final gather often bottleneck...

Ambient Occlusion

- A "hack" to create more realistic ambient illumination cheaply
- Assume light from everywhere is partially blocked by local objects
  - At a point on the surface cast rays at random
  - Ambient term is proportional to percent of rays that hit nothing
  - Weight average by cosine of angle with normal
  - Take into account how far before occluded

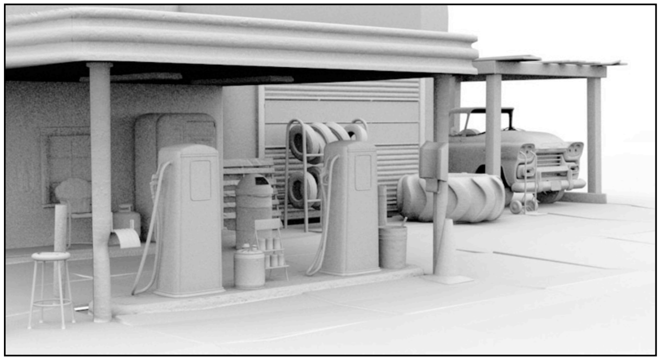
Ambient Occlusion



Ambient Occlusion

Diffuse Only | Ambient Occlusion | Combined

Ambient Occlusion



nVidia Gelato Demo Image