Today

- **Radiometry: measuring light**
  - Local Illumination and Raytracing were discussed in an *ad hoc* fashion
  - Proper discussion requires proper units
  - Not just pretty pictures... but correct pictures
Matching Reality
Matching Reality

Photo

Rendered

Cornell Box Comparison
Cornell Program of Computer Graphics
Units

- **Light energy**
  - Really power not energy is what we measure
  - Joules / second (J/s) = Watts (W)

- **Spectral energy density**
  - power per unit spectrum interval
  - Watts / nano-meter (W/nm)
  - Properly done as function over spectrum
  - Often just sampled for RGB

- Often we assume people know we’re talking about S.E.D. and just say E...
Irradiance

- Total light striking surface from all directions
  - Only meaningful w.r.t. a surface
  - Power per square meter ($W/m^2$)
  - Really S.E.D. per square meter ($W/m^2/nm$)
  - Not all directions sum the same because of foreshortening
Radiant Exitance

- Total light *leaving* surface over all directions
  - Only meaningful w.r.t. a surface
  - Power per square meter ($\text{W/m}^2$)
  - Really S.E.D. per square meter ($\text{W/m}^2/\text{nm}$)
  - Also called Radiosity
  - Sum over all directions $\Rightarrow$ same in all directions

$\text{W/m}^2$
Solid Angles

- **Regular angles measured in radians**
  - Measured by arc-length on unit circle \([0..2\pi]\)

- **Solid angles measured in steradians**
  - Measured by area on unit sphere \([0..4\pi]\)
  - Not necessarily little round pieces...
Radiance

- Light energy passing through a point in space in a given direction
  - Energy per steradian per square meter (W/m²/sr)
  - S.E.D. per steradian per square meter (W/m²/sr/nm)
- Constant along straight lines in free space
Radiance

- Near surfaces, differentiate between
  - Radiance from the surface (surface radiance)
  - Radiance from other things (field radiance)

\[ L_s \quad L_f \]
Light Fields

- The radiance at every point in space, direction, and frequency: 6D function
- Collapse frequency to RGB, and assume free space: 4D function
- Sample and record it over some volume
Light Fields

Levoy and Hanrahan, SIGGRAPH 1996
Light Fields

Levoy and Hanrahan, SIGGRAPH 1996
Light Fields

Michelangelo’s Statue of Night
From the Digital Michelangelo Project
Computing Irradiance

- Integrate incoming radiance (field radiance) over all direction
  - Take into account foreshortening

\[ H = \int_{\Omega} L_f(k) \cos(\theta) d\sigma \]

\[ H = \int_{0}^{2\pi} \int_{0}^{\pi/2} L_f(\theta, \phi) \cos(\theta) \sin(\theta) \ d\theta \ d\phi \]
Revisiting The BRDF

- How much light from direction A goes out in direction B

- Now we can talk about units:
  - BRDF is ratio of foreshortened field radiance to surface radiance

\[
\rho(\theta_i, \theta_o) = \frac{L_s(\theta_o)}{L_f(\theta_i) \cos(\hat{n}\theta)}
\]

We left out frequency dependance here...

Also note for perfect Lambertian reflector with constant BRDF

\[
\rho = \frac{1}{\pi}
\]
The Rendering Equation

Total light going out in some direction is given by an integral over all incoming directions:

\[ L_s(k_o) = \int_{\Omega} \rho(k_o, k_i) L_f(k_i) \cos(\theta) d\sigma \]

Note, this is recursive (my \( L_f \) is another’s \( L_s \))
The Rendering Equation

- We can rewrite explicitly in terms of $L_s$

$$L_s(k_o) = \int_{\Omega} \rho(k_o, k_i)L_f(k_i) \cos(\theta_i)d\sigma$$

$$L_s(k_o, x) = \int_{S} \rho(k_o, k_i)L_s(x - x', x') \cos(\theta_i) \cos(\hat{n}'(x - x')) \delta(x, x') \frac{1}{||x - x'||^2} \, dx'$$

Consider what ray tracing was doing....
Light Paths

- Many paths from light to eye
- Characterize by the types of bounces
  - Begin at light
  - End at eye
  - “Specular” bounces
  - “Diffuse” bounces
Light Paths

- **Describe paths using strings**
  - LDE, LDSE, LSE, *etc.*

- **Describe types of paths with regular expressions**
  - \(L\{D|S\}\)*E
  - \(L\{D|S\}\)S*E
  - \(L\{D|S\}\)E
  - LD*E

  - Visible paths
  - Standard raytracing
  - Local illumination
  - Radiosity method
    - (have not talked about yet)