

# CS-184: Computer Graphics

## Lecture #9: Scan Conversion

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V2008-F-09-1.0

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# Today

- 2D Scan Conversion

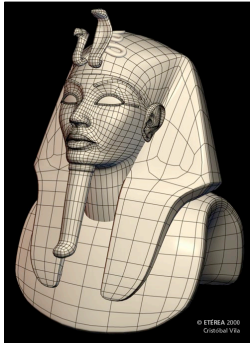
- Drawing Lines
- Drawing Curves
- Filled Polygons
- Filling Algorithms

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## Drawing a Line

- Basically, its easy... but for the details
- Lines are a basic primitive that needs to be done well...



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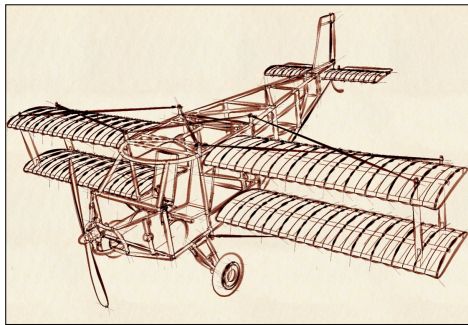
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## Drawing a Line

- Basically, its easy... but for the details
- Lines are a basic primitive that needs to be done well...



From "A Procedural Approach to Style for NPR Line Drawing from 3D models,"  
by Grabli, Durand, Turquin, Sillion

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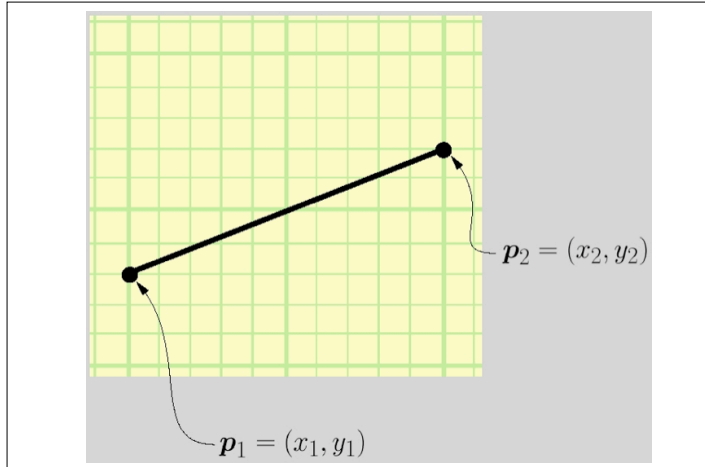
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## Drawing a Line



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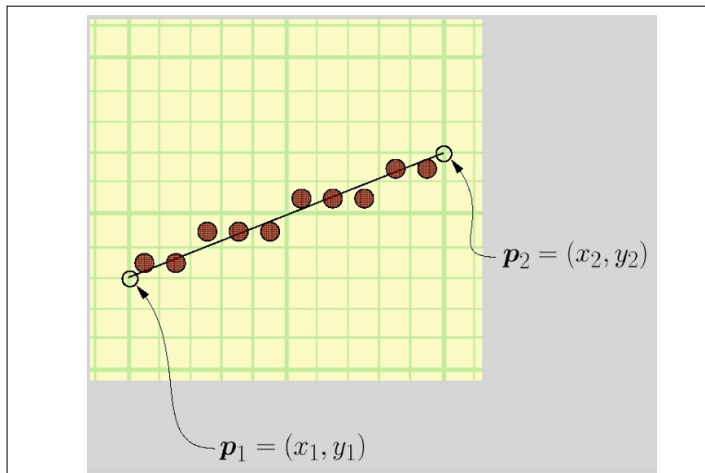
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## Drawing a Line



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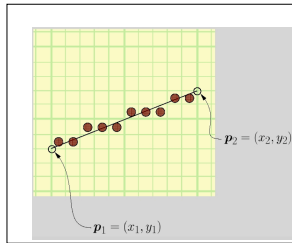
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## Drawing a Line

- Some things to consider
  - How thick are lines?
  - How should they join up?
  - Which pixels are the right ones?



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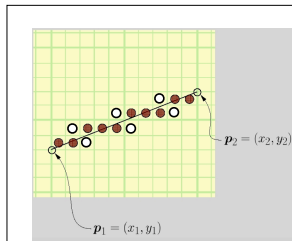
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## Drawing a Line

- Some things to consider
  - How thick are lines?
  - How should they join up?
  - Which pixels are the right ones?

For example:



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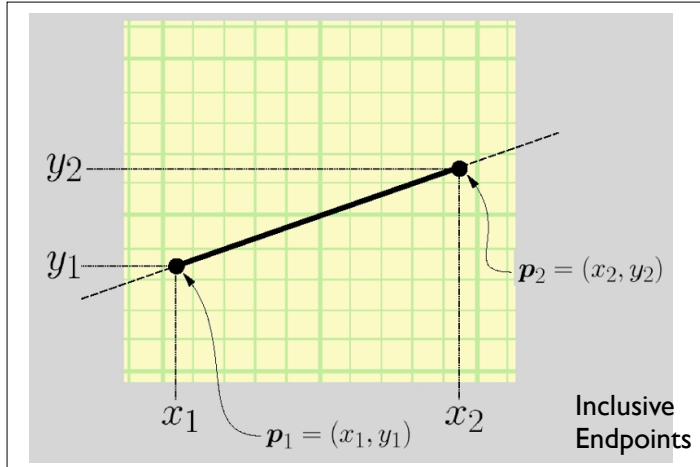
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## Drawing a Line



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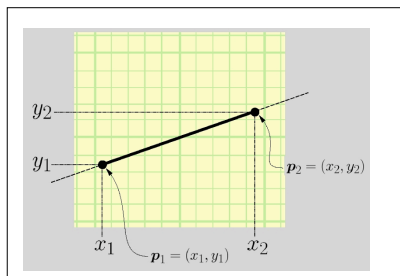
8

## Drawing a Line

$$y = m \cdot x + b, x \in [x_1, x_2]$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

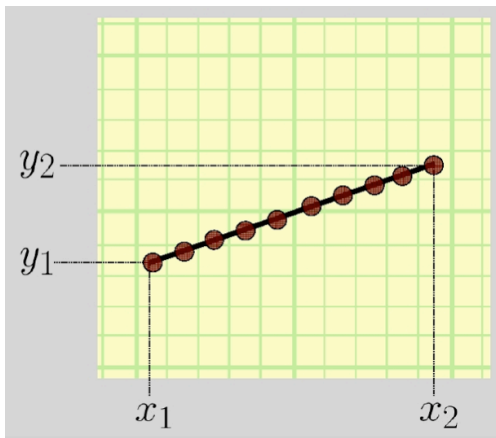
$$b = y_1 - m \cdot x_1$$



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## Drawing a Line

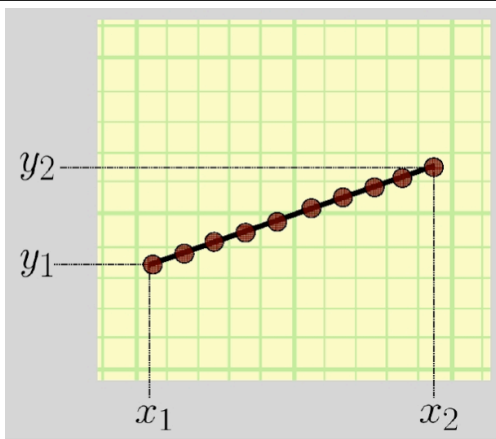


$$\Delta x = 1$$
$$\Delta y = m \cdot \Delta x$$

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## Drawing a Line



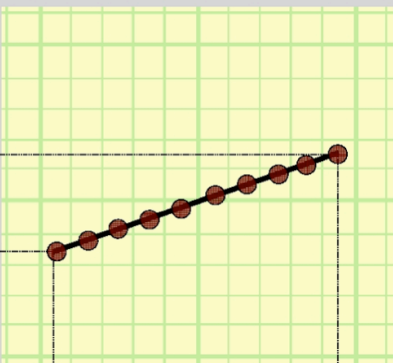
$$\Delta x = 1$$
$$\Delta y = m \cdot \Delta x$$

```
x=x1
y=y1
while(x<=x2)
  plot(x,y)
  x++
  y+=Dy
```

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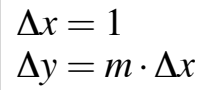
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# Drawing a Line



$\Delta x = 1$   
 $\Delta y = m \cdot \Delta x$

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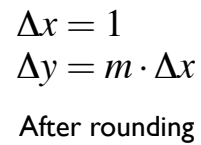
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[illegible]

# Drawing a Line

$\Delta x = 1$   
 $\Delta y = m \cdot \Delta x$   
After rounding

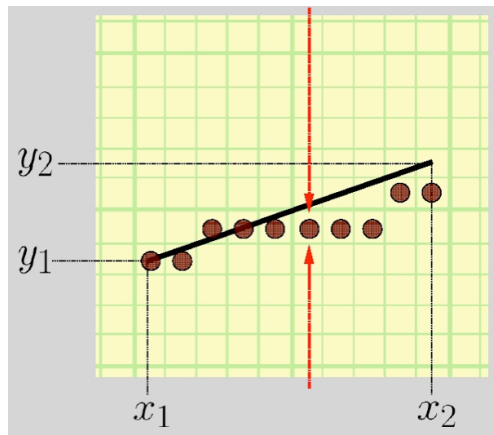
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This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

## Drawing a Line



$$\Delta x = 1$$

$$\Delta y = m \cdot \Delta x$$

$$y += \Delta y$$

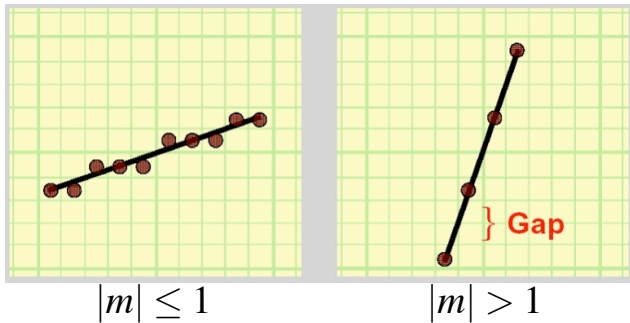
Accumulation of  
roundoff errors

How slow is float-  
to-int conversion?

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## Drawing a Line



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## Drawing a Line

```
void drawLine-Error1(int x1,x2, int y1,y2)

    float m = float(y2-y1)/(x2-x1)
    int x = x1
    float y = y1

    while (x <= x2)

        setPixel(x,round(y),PIXEL_ON)

        x += 1
        y += m
```

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## Drawing a Line

```
void drawLine-Error1(int x1,x2, int y1,y2)

    float m = float(y2-y1)/(x2-x1)
    int x = x1
    float y = y1

    while (x <= x2)

        setPixel(x,round(y),PIXEL_ON)

        x += 1
        y += m
```

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## Drawing a Line

```
void drawLine-Error1(int x1,x2, int y1,y2)

float m = float(y2-y1)/(x2-x1)
int x = x1
float y = y1

while (x <= x2)

    setPixel(x,round(y),PIXEL_ON)

    x += 1
    y += m
```

Not exact math

Accumulates errors

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## Drawing a Line

```
void drawLine-Error2(int x1,x2, int y1,y2)

float m = float(y2-y1)/(x2-x1)
int x = x1
int y = y1
float e = 0.0

while (x <= x2)

    setPixel(x,y,PIXEL_ON)

    x += 1
    e += m
    if (e >= 0.5)
        y+=1
        e-=1.0
```

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## Drawing a Line

```
void drawLine-Error2(int x1,x2, int y1,y2)

float m = float(y2-y1)/(x2-x1)
int x = x1
int y = y1
float e = 0.0

while (x <= x2)

    setPixel(x,y,PIXEL_ON)

    x += 1
    e += m
    if (e >= 0.5)
        y+=1
        e-=1.0
```

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## Drawing a Line

```
void drawLine-Error3(int x1,x2, int y1,y2)

int x = x1
int y = y1
float e = -0.5

while (x <= x2)

    setPixel(x,y,PIXEL_ON)

    x += 1
    e += float(y2-y1)/(x2-x1)
    if (e >= 0.0)
        y+=1
        e-=1.0
```

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## Drawing a Line

```
void drawLine-Error4(int x1,x2, int y1,y2)

    int x = x1
    int y = y1
    float e = -0.5*(x2-x1)          // was -0.5

    while (x <= x2)

        setPixel(x,y,PIXEL_ON)

        x += 1
        e += y2-y1                  // was /(x2-x1)
        if (e >= 0.0)                // no change
            y+=1
            e-=(x2-x1)              // was 1.0
```

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## Drawing a Line

```
void drawLine-Error5(int x1,x2, int y1,y2)

    int x = x1
    int y = y1
    int e = -(x2-x1)                // removed *0.5

    while (x <= x2)

        setPixel(x,y,PIXEL_ON)

        x += 1
        e += 2*(y2-y1)              // added 2*
        if (e >= 0.0)                // no change
            y+=1
            e-=2*(x2-x1)            // added 2*
```

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## Drawing a Line

```
void drawLine-Bresenham(int x1,x2, int y1,y2)
```

```
int x = x1  
int y = y1  
int e = -(x2-x1)
```

```
while (x <= x2)
```

```
    setPixel(x,y,PIXEL_ON)
```

```
    x += 1
```

```
    e += 2*(y2-y1)
```

```
    if (e >= 0.0)
```

```
        y+=1
```

```
        e-=2*(x2-x1)
```

Faster  
Not wrong

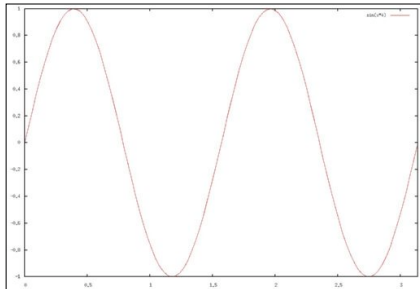
$$|m| \leq 1$$

$$x_1 \leq x_2$$

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## Drawing Curves



$$y = f(x)$$

Only one value of  $y$  for each value of  $x$ ...

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## Drawing Curves

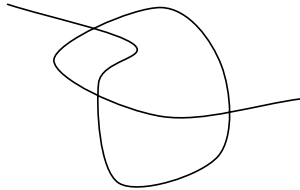
- Parametric curves

- Both  $x$  and  $y$  are a function of some third parameter

$$x = f(u)$$
$$y = f(u)$$

$$\mathbf{x} = \mathbf{f}(u)$$

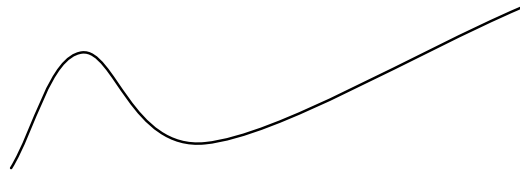
$$u \in [u_0 \dots u_1]$$



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## Drawing Curves



$$\mathbf{x} = \mathbf{f}(u)$$

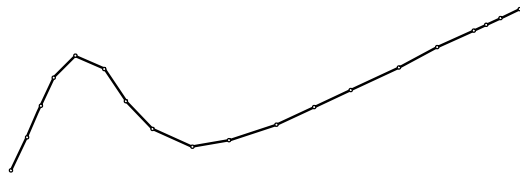
$$u \in [u_0 \dots u_1]$$

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## Drawing Curves

- Draw curves by drawing line segments
  - Must take care in computing end points for lines
  - How long should each line segment be?



$$\mathbf{x} = \mathbf{f}(u) \quad u \in [u_0 \dots u_1]$$

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## Drawing Curves

- Draw curves by drawing line segments
  - Must take care in computing end points for lines
  - How long should each line segment be?
  - Variable spaced points



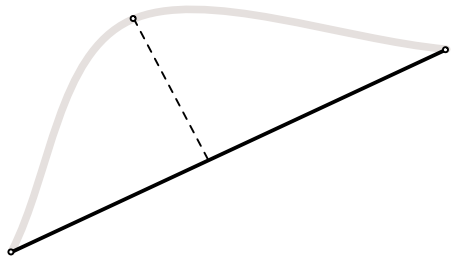
$$\mathbf{x} = \mathbf{f}(u) \quad u \in [u_0 \dots u_1]$$

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## Drawing Curves

- Midpoint-test subdivision



$$|\mathbf{f}(u_{mid}) - \mathbf{l}(0.5)|$$

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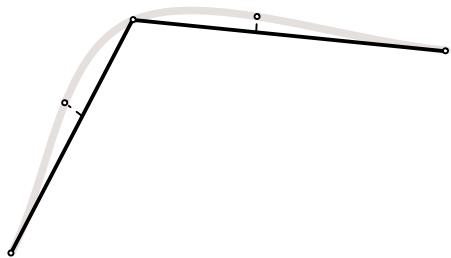
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## Drawing Curves

- Midpoint-test subdivision



$$|\mathbf{f}(u_{mid}) - \mathbf{l}(0.5)|$$

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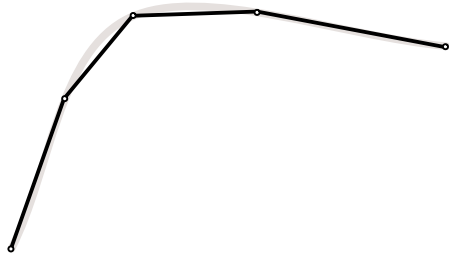
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## Drawing Curves

- Midpoint-test subdivision



$$|\mathbf{f}(u_{mid}) - \mathbf{l}(0.5)|$$

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## Drawing Curves

- Midpoint-test subdivision

- Not perfect
- We need more information for a guarantee...

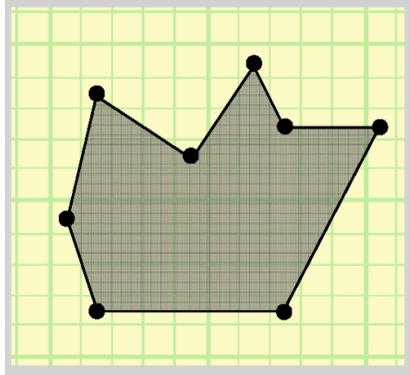


$$|\mathbf{f}(u_{mid}) - \mathbf{l}(0.5)|$$

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## Filled Polygons



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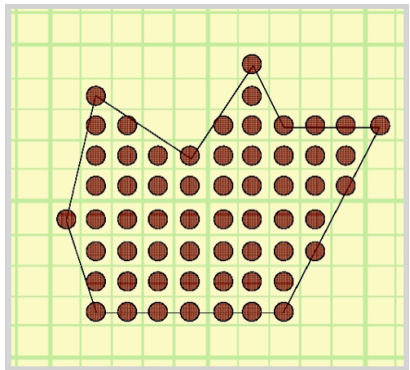
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## Filled Polygons



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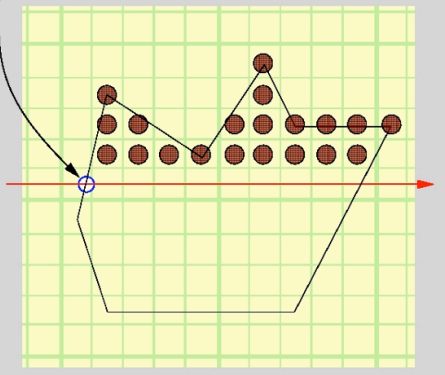
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## Filled Polygons

Toggle inside/outside flag to "INSIDE"



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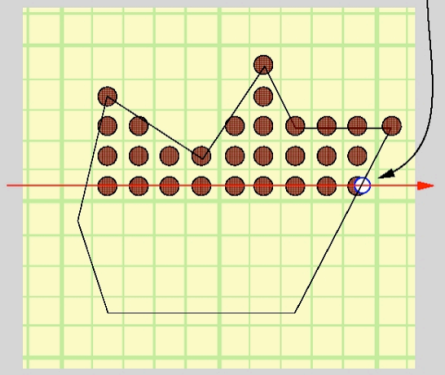
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## Filled Polygons

Toggle inside/outside flag to "OUTSIDE"



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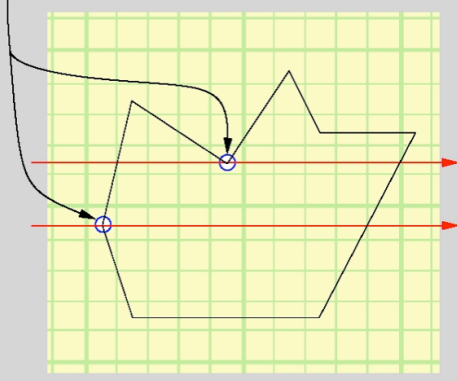
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## Filled Polygons

What happens at these locations?



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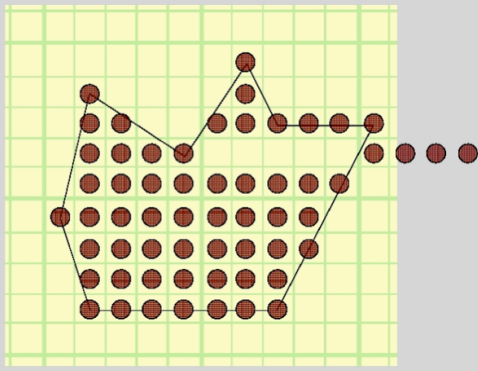
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## Filled Polygons

If we count ONCE...



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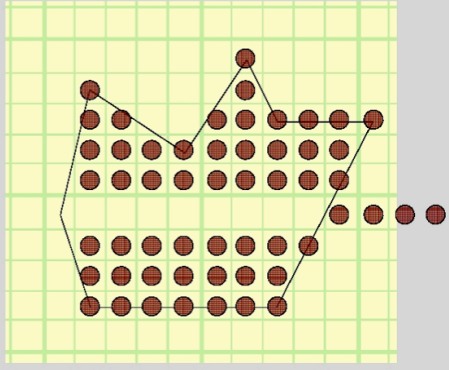
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## Filled Polygons

If we count TWICE...



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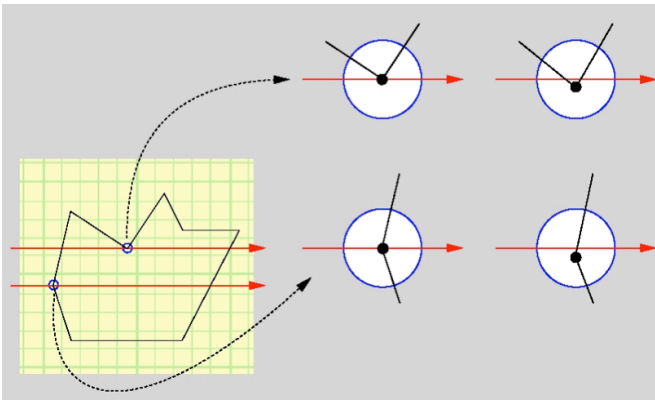
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## Filled Polygons

Treat (scan  $y$  = vertex  $y$ ) as (scan  $y$  > vertex  $y$ )



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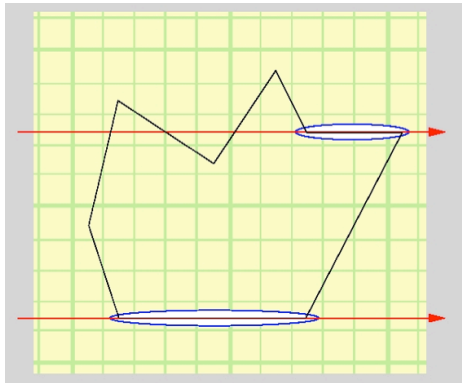
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## Filled Polygons

Horizontal edges



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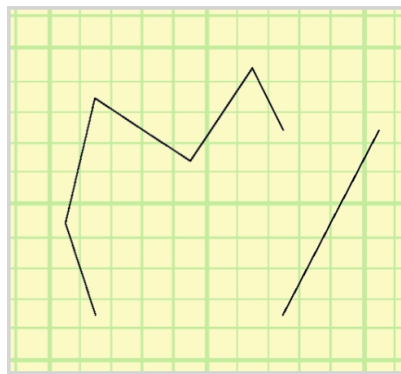
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## Filled Polygons

Horizontal edges



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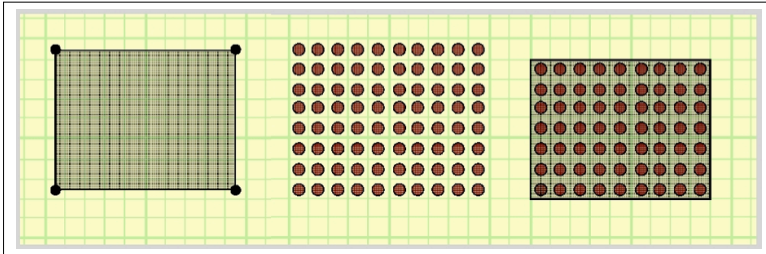
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## Filled Polygons

- “Equality Removal” applies to all vertices
- Both  $x$  and  $y$  coordinates

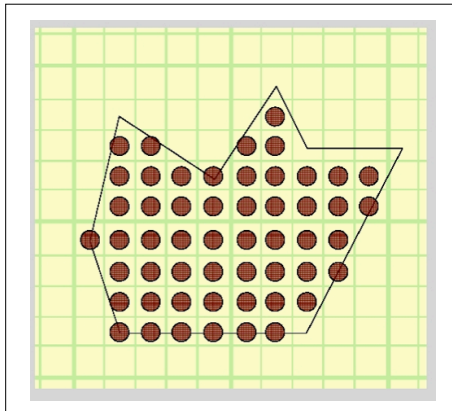


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## Filled Polygons

- Final result:

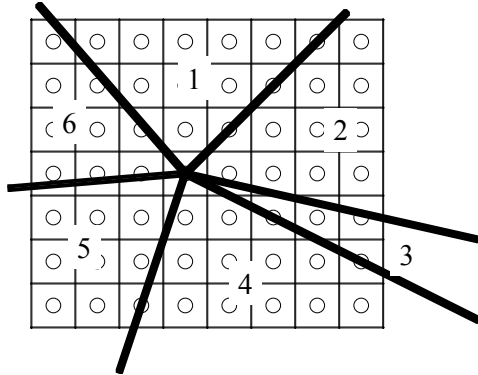


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## Filled Polygons

- Who does this pixel belong to?



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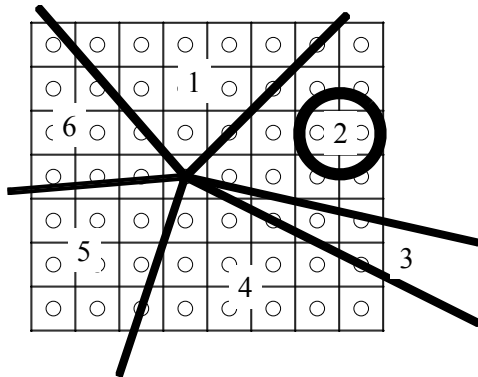
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## Filled Polygons

- Who does this pixel belong to?



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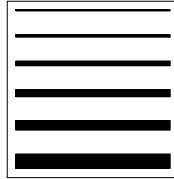
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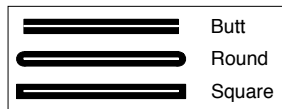


## Drawing a Line

- How thick?



- Ends?



42

42

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

## Drawing a Line

- Joining?



Ugly

Bevel

Round

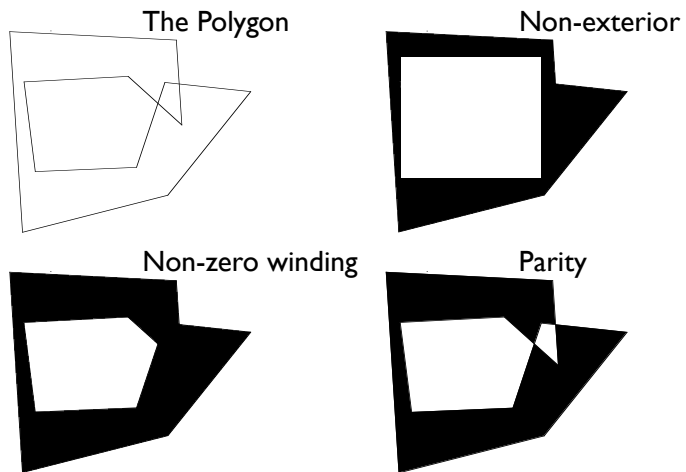
Miter

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[illegible]

## Inside/Outside Testing



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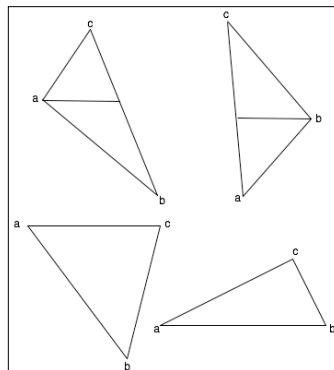
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## Optimize for Triangles

- Split triangle into two parts
  - Two edges per part
  - Y-span is monotonic
- For each row
  - Interpolate span
- Interpolate barycentric coordinates



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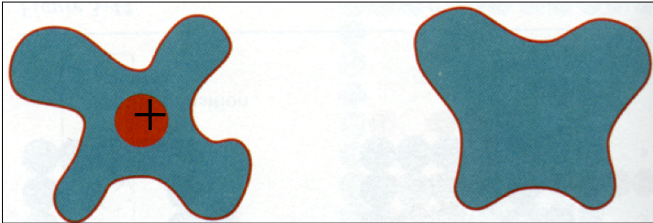
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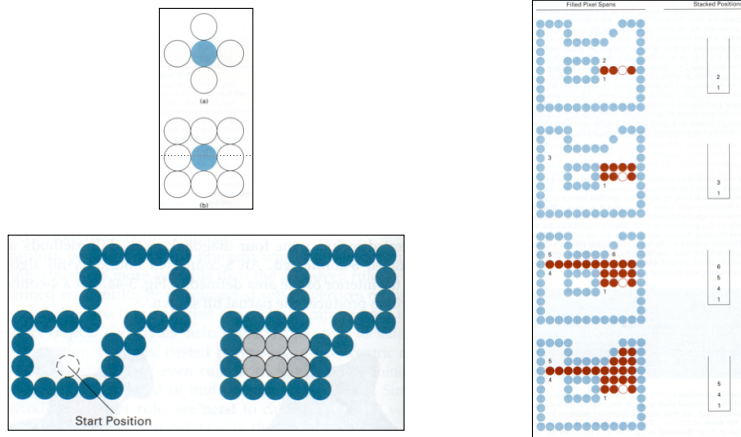
## Flood Fill



46

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## Flood Fill



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47