CS-184: Computer Graphics

Lecture #22: Spring and Mass systems

Prof. James O'Brien
University of California, Berkeley
Today

- Spring and Mass systems
  - Distance springs
  - Spring dampers
  - Edge springs
A Simple Spring

- **Ideal zero-length spring**

  \[ f_{a \rightarrow b} = k_s (b - a) \]

  \[ f_{b \rightarrow a} = -f_{a \rightarrow b} \]

- **Force pulls points together**

- **Strength proportional to distance**
A Simple Spring

○ Energy potential

\[ E = \frac{1}{2} k_S (b - a) \cdot (b - a) \]

\[ f_{a \rightarrow b} = k_S (b - a) \]

\[ f_{b \rightarrow a} = -f_{a \rightarrow b} \]

\[ f_a = -\nabla_a E = - \left[ \frac{\partial E}{\partial a_x}, \frac{\partial E}{\partial a_y}, \frac{\partial E}{\partial a_z} \right] \]
A Simple Spring

- Energy potential: kinetic vs elastic

\[ E = \frac{1}{2} k_s (b - a) \cdot (b - a) \]

\[ E = \frac{1}{2} m (\dot{b} - \dot{a}) \cdot (\dot{b} - \dot{a}) \]
Non-Zero Length Springs

\[ f_{a \to b} = k_s \frac{b - a}{||b - a||} \left( ||b - a|| - l \right) \]

Rest length

\[ E = k_s \left( ||b - a|| - l \right)^2 \]
Comments on Springs

- **Springs with zero rest length are linear**
- **Springs with non-zero rest length are non-linear**
  - Force *magnitude* linear w/ displacement (from rest length)
  - Force direction is non-linear
- **Singularity at** $\|b - a\| = 0$
Damping

○ “Mass proportional” damping
\[ f = -k_d \dot{a} \]

○ Behaves like viscous drag on all motion

○ Consider a pair of masses connected by a spring
  ○ How to model rusty vs oiled spring
  ○ Should internal damping slow group motion of the pair?

○ Can help stability... up to a point
“Stiffness proportional” damping

\[ f_a = -kd \frac{b - a}{\|b - a\|^2} (b - a) \cdot (\dot{b} - \dot{a}) \]

- Behaves viscous drag on change in spring length
- Consider a pair of masses connected by a spring
  - How to model rusty vs oiled spring
  - Should internal damping slow group motion of the pair?
Spring Constants

- Two ways to model a single spring

\[ l \]
\[ \Delta l \]
Spring Constants

- Constant $k_s$ gives inconsistent results with different discretizations.
- Change in length is not what we want to measure.
- Strain: change in length as fraction of original length

$$\varepsilon = \frac{\Delta l}{l_0}$$

Nice and simple for 1D...
Structures from Springs

- Sheets
- Blocks
- Others
Structures from Springs

- They behave like what they are (obviously!)

This structure will not resist shearing

This structure will not resist out-of-plane bending either...
Structures from Springs

- They behave like what they are (obviously!)

This structure will resist shearing but has anisotropic bias

This structure still will not resist out-of-plane bending
Structures from Springs

- They behave like what they are (obviously!)

This structure will resist shearing
Less bias
Interference between spring sets

This structure still will not resist out-of-plane bending
Structures from Springs

- They behave like what they are (obviously!)

- This structure will resist shearing
  - Less bias
  - Interference between spring sets

- This structure will resist out-of-plane bending
  - Interference between spring sets
  - Odd behavior

- How do we set spring constants?
Edge Springs

$u_1 = |E| \frac{N_1}{|N_1|^2}$  $u_2 = |E| \frac{N_2}{|N_2|^2}$

$u_3 = \frac{(x_1 - x_4) \cdot E}{|E|} \frac{N_1}{|N_1|^2} + \frac{(x_2 - x_4) \cdot E}{|E|} \frac{N_2}{|N_2|^2}$

$u_4 = -\frac{(x_1 - x_3) \cdot E}{|E|} \frac{N_1}{|N_1|^2} - \frac{(x_2 - x_3) \cdot E}{|E|} \frac{N_2}{|N_2|^2}$

$F_i^e = k^e \frac{|E|^2}{|N_1| + |N_2|} \sin(\theta/2) u_i$

From Bridson et al., 2003, also see Grinspun et al., 2003
Suggested Reading

- Physically Based Modeling: Principles and Practice
  - Andy Witkin and David Baraff
- Bridson, Marino, and Fedkiw, "Simulation of Clothing with Folds and Wrinkles," SCA 2003