Today

- Raytracing
  - Shadows and direct lighting
  - Reflection and refraction
  - Antialiasing, motion blur, soft shadows, and depth of field

- Intersection Tests
  - Ray-primitive
  - Sub-linear tests
Raytracing Assignment

Hint: (dueDate - startDate) \propto pain^{-1}

\[ R_x = -x + 2(x \cdot y) \]

\[ V(t) = E + t(P - E) \]

\[ k_1 \cdot I + k_2 \cdot I_{\max} (I \cdot \hat{N}, 0) + k_3 \cdot I_{\max} (R \cdot \hat{V}, 0) \]

\[ \hat{N} \]

\[ \hat{V} \]

HAAAAAAAAAAGGGHHHH...
Light in an Environment

Lady writing a Letter with her Maid
National Gallery of Ireland, Dublin
Johannes Vermeer, 1670
Global Illumination Effects

PCKTWTH
Kevin Odhner
POV-Ray
Global Illumination Effects

A Philco 6Z4 Vacuum Tube
Steve Anger
POV-Ray
Global Illumination Effects

Caustic Sphere
Henrik Jensen
(refraction caustic)
Global Illumination Effects

Caustic Ring
Henrik Jensen
(reflection caustic)
Global Illumination Effects

Sphere Flake
Henrik Jensen
Early Raytracing

Turner Whitted
Raytracing

- **Scan conversion**
  - $3D \rightarrow 2D \rightarrow \text{Image}$
  - Based on transforming geometry

- **Raytracing**
  - $3D \rightarrow \text{Image}$
  - Geometric reasoning about light rays
Raytracing

Eye, view plane section, and scene
Raytracing

Launch ray from eye through pixel, see what it hits
Raytracing

Compute color and fill-in the pixel
Raytracing

- Basic tasks
  - Build a ray
  - Figure out what a ray hits
  - Compute shading
Building Eye Rays

- Rectilinear image plane build from four points

\[ P = u \ (v_{LL} + (1 - v)UL) + (1 - u) (v_{LR} + (1 - v)UR) \]
Building Eye Rays

- Nonlinear projections
  - Non-planar projection surface
  - Variable eye location
Examples

Multiple-Center-of-Projection Images
P. Rademacher and G. Bishop
SIGGRAPH 1998
Examples

Spherical and Cylindrical Projections
Ben Kreunen
From Big Ben's Panorama Tutorials
Building Eye Rays

- Ray equation

$$R(t) = E + t(P - E)$$

$$t \in [1 \ldots +\infty]$$

- Through eye at $$t = 0$$
- At pixel center at $$t = 1$$
Detect shadow by rays to light source

\[ R(t) = S + t(L - S) \]

\( t \in [\varepsilon \ldots 1) \)
Shadow Rays

- **Test for occluder**
  - No occluder, shade normally (e.g. Phong model)
  - Yes occluder, skip light (don’t skip ambient)

- **Self shadowing**
  - Add shadow bias
  - Test object ID

![Self-shadowing](image1)
![Correct](image2)
Reflection Rays

- **Recursive shading**
  - Ray bounces off object
  - Treat bounce rays (mostly) like eye rays
  - Shade bounce ray and return color
    - Shadow rays
    - Recursive reflections
  - Add color to shading at original point
    - Specular or separate reflection coefficient

\[ R(t) = S + tB \quad t \in [\varepsilon \ldots + \infty) \]
Reflection Rays

- **Recursion Depth**
  - Truncate at fixed number of bounces
  - Multiplier less than J.N.D.
Refracted Rays

- Transparent materials bend light
  - Snell’s Law \( \frac{n_i}{n_t} = \frac{\sin \theta_t}{\sin \theta_i} \) (see clever formula in text...)
  
  \( \sin \theta_t > 1 \iff \text{Total (internal) reflection} \)
Refracted Rays

- Coefficient on transmitted ray depends on $\theta$
  - Schlick approximation to Fresnel Equations
    \[ k_t(\theta_i) = k_0 + (1 - k_0)(1 - \cos \theta_i)^5 \]
    \[ k_0 = \left( \frac{n_t - 1}{n_t + 1} \right)^2 \]
  - Attenuation
    - Wavelength (color) dependant
    - Exponential with distance
Refracted Rays

O’Brien and Hodgins, SIGGRAPH 1999
Anti-Aliasing

- Boolean on/off for pixels causes problems
  - Consider scan conversion algorithm:
    - Compare to casting a ray through each pixel center
  - Recall Nyquist Theorem
    - \( \text{Sampling rate} \geq \text{twice highest frequency} \)
Anti-Aliasing

- Desired solution of an integral over pixel
“Distributed” Raytracing

- Send multiple rays through each pixel
- Average results together
- Jittering trades aliasing for noise

One Sample  5x5 Grid  5x5 Jittered Grid
“Distributed” Raytracing

- Use multiple rays for reflection and refraction
  - At each bounce send out many extra rays
  - Quasi-random directions
  - Use BRDF (or Phong approximation) for weights

- How many rays?
Soft Shadows

- Soft shadows result from non-point lights
- Some part of light visible, some other part occluded
Soft Shadows

- Distribute shadow rays over light surface

All shadow rays go through

No shadow rays go through

Some shadow rays go through

Figure from S. Chenney
Motion Blur

- Distribute rays over time
  - More when we talk about animation...

Pool Bails
Tom Porter
RenderMan
Depth of Field

- Distribute rays over a lens assembly
Depth of Field

No DoF

Jittered rays for DoF

Multiple images for DoF

More rays

Even more rays
Other Lens Effects
Ray -vs- Sphere Test

- Ray equation: \( R(t) = A + tD \)
- Implicit equation for sphere: \( |X - C|^2 - r^2 = 0 \)
- Combine:
  \[
  |R(t) - C|^2 - r^2 = 0 \\
  |A + tD - C|^2 - r^2 = 0 
  \]
- Quadratic equation in \( t \)
Ray -vs- Sphere Test

Two solutions

One solution

Imaginary
Ray -vs- Triangle

- Ray equation: \( R(t) = A + tD \)
- Triangle in barycentric coordinates:
  \[
  X(\beta, \gamma) = V_1 + \beta(V_2 - V_1) + \gamma(V_3 - V_1)
  \]
- Combine:
  \[
  V_1 + \beta(V_2 - V_1) + \gamma(V_3 - V_1) = A + tD
  \]
- Solve for \( \beta, \gamma, \) and \( t \)
  - 3 equations 3 unknowns
  - Beware divide by near-zero
  - Check ranges