CS-184: Computer Graphics

Lecture #23: Spring and Mass systems

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Today

- Spring and Mass systems
  - Distance springs
  - Spring dampers
  - Edge springs
A Simple Spring

- Ideal zero-length spring
  \[ f_{a\rightarrow b} = k_s(b - a) \]
  \[ f_{b\rightarrow a} = -f_{a\rightarrow b} \]
- Force pulls points together
- Strength proportional to distance

A Simple Spring

- Energy potential
  \[ E = \frac{1}{2} k_s (b - a) \cdot (b - a) \]
  \[ f_{a\rightarrow b} = k_s (b - a) \]
  \[ f_{b\rightarrow a} = -f_{a\rightarrow b} \]
  \[ f_a = -\nabla_a E = - \left[ \frac{\partial E}{\partial x}, \frac{\partial E}{\partial y}, \frac{\partial E}{\partial z} \right] \]
A Simple Spring

- Energy potential: kinetic vs elastic

\[
E = \frac{1}{2} k_s (b - a) \cdot (b - a)
\]

\[
E = \frac{1}{2} m (\dot{b} - \dot{a}) \cdot (\dot{b} - \dot{a})
\]

Non-Zero Length Springs

\[
f_{a \rightarrow b} = k_s \frac{b - a}{||b - a||} (||b - a|| - l)
\]

\[
E = k_s (||b - a|| - l)^2
\]
Comments on Springs

- Springs with zero rest length are linear
- Springs with non-zero rest length are nonlinear
  - Force *magnitude* linear w/ displacement (from rest length)
  - Force direction is non-linear
  - Singularity at $||b - a|| = 0$

Damping

- “Mass proportional” damping
  \[ f = \alpha \ddot{a} \]
  - Behaves like viscous drag on all motion
  - Consider a pair of masses connected by a spring
    - How to model rusty vs oiled spring
    - Should internal damping slow group motion of the pair?
  - Can help stability... up to a point
Damping

- “Stiffness proportional” damping

\[ f_a = -k_d \frac{b - a}{||b - a||^2} (b - a) \cdot (\dot{b} - \dot{a}) \]

- Behaves viscous drag on change in spring length
- Consider a pair of masses connected by a spring
  - How to model rusty vs oiled spring
  - Should internal damping slow group motion of the pair?

Spring Constants

- Two ways to model a single spring
Spring Constants

- Constant $k_S$ gives inconsistent results with different discretizations.
- Change in length is not what we want to measure.
- Strain: change in length as fraction of original length

$$\epsilon = \frac{\Delta l}{l_0}$$  Nice and simple for 1D...

Structures from Springs

- Sheets

- Blocks

- Others
Structures from Springs

- They behave like what they are (obviously!)

This structure will not resist shearing

This structure will not resist out-of-plane bending either...

Structures from Springs

- They behave like what they are (obviously!)

This structure will resist shearing but has anisotropic bias

This structure still will not resist out-of-plane bending
Structures from Springs

- They behave like what they are (obviously!)

This structure will resist shearing
Less bias
Interference between spring sets

This structure still will not resist out-of-plane bending

How do we set spring constants?
**Edge Springs**

\[ u_1 = \frac{|E|}{|N_1|} \quad u_2 = \frac{|E|}{|N_2|} \]

\[ u_3 = \frac{(x_1 - x_3) \cdot E}{|E|} \frac{N_1}{|N_1|^2} + \frac{(x_2 - x_3) \cdot E}{|E|} \frac{N_2}{|N_2|^2} \]

\[ u_4 = -\frac{(x_1 - x_3) \cdot E}{|E|} \frac{N_1}{|N_1|^2} - \frac{(x_2 - x_3) \cdot E}{|E|} \frac{N_2}{|N_2|^2} \]

\[ F_i^e = k^e \frac{|E|^2}{|N_1| + |N_2|} \sin(\theta/2) u_i \]

From Bridson et al., 2003, also see Grinspun et al., 2003

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**Suggested Reading**

- Physically Based Modeling: Principles and Practice
  - Andy Witkin and David Baraff
- Bridson, Marino, and Fedkiw, "Simulation of Clothing with Folds and Wrinkles," SCA 2003
- O'Brien and Hodgins, "Graphical Modeling and Animation of Brittle Fracture," SIGGRAPH 99