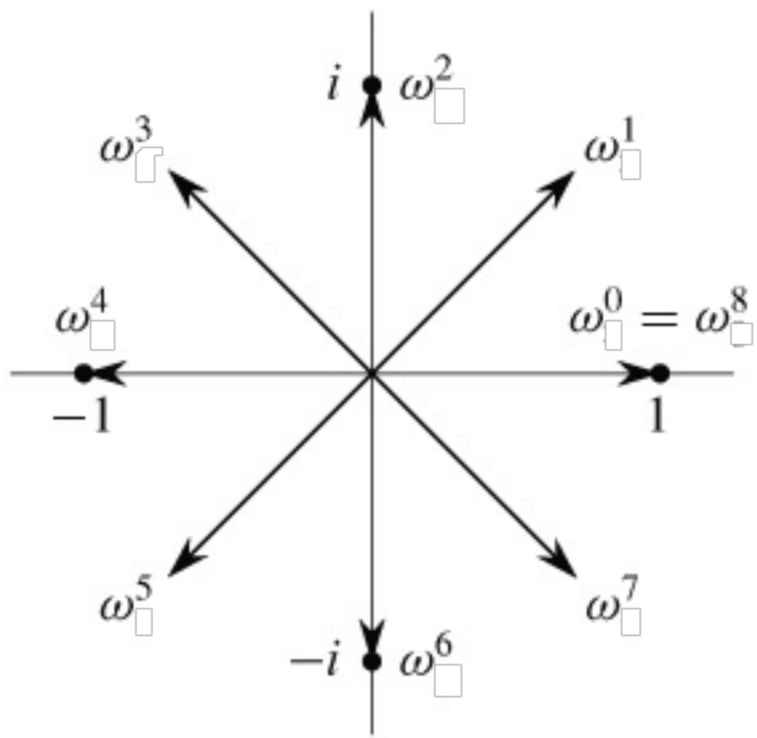
FFT Example

Evaluate the polynomial:

$P(x) = 1+3x+5x^2+7x^3+8x^4+6x^5+3x^6+2x^7$

at points $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7$ where $\omega^k=e^{i2\pi k/8}$, k = 0, 1, 2 … 7



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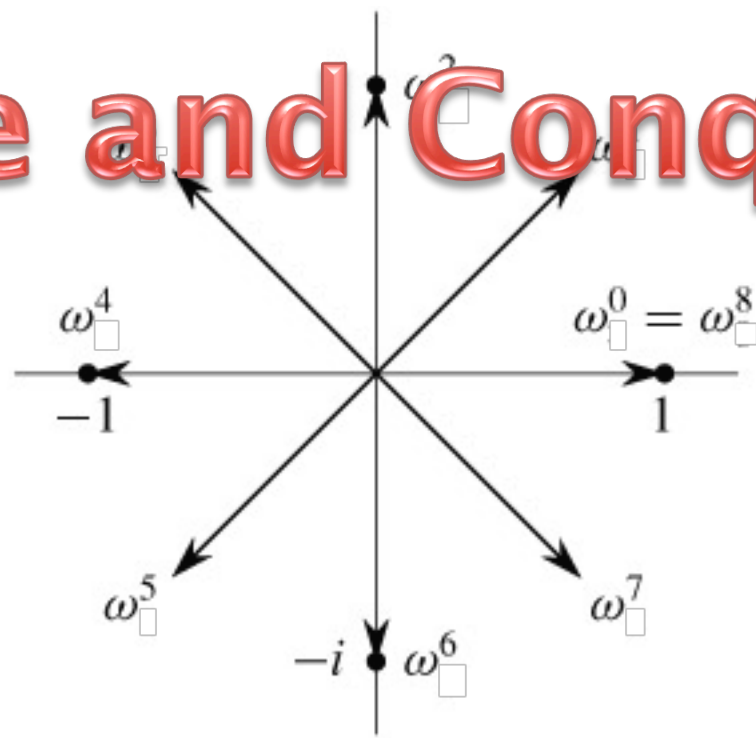
Divide and Conquer !!!



1^{st} step: Divide

Evaluate the polynomial:

$P(x) = 1+3x+5x^2+7x^3+8x^4+6x^5+3x^6+2x^7$

at points ω^0, ω^1, ω^2, ω^3, ω^4, ω^5, ω^6, ω^7 where $\omega^k=e^{i2\pi k/8}$, k = 0, 1, 2 … 7

Recursively solve the following problems and combine the solutions.

1) Evaluate $P_e(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, k = 0, 1, 2, 3

2) Evaluate $P_o(x) = 3+7x+6x^2+2x^3$ at points $\omega^k=e^{i2\pi k/4}$, k = 0, 1, 2, 3

1^{st} step: Divide

Evaluate the polynomial:

$P(x) = 1+3x+5x^2+7x^3+8x^4+6x^5+3x^6+2x^7$

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1) Evaluate $P_e(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, k = 0, 1, 2, 3

   Note: $P_e(x^2) = 1+5x^2+8x^4+3x^6$

2) Evaluate $P_o(x) = 3+7x+6x^2+2x^3$ at points $\omega^k=e^{i2\pi k/4}$, k = 0, 1, 2, 3

   Note: $xP_o(x^2) = 3x+7x^3+6x^5+2x^7$

1^{st} step: Divide

Evaluate the polynomial:

$P(x) = 1+3x+5x^2+7x^3+8x^4+6x^5+3x^6+2x^7$

at points $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7$ where $\omega^k=e^{i2\pi k/8}$, k = 0, 1, 2 … 7

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   Why is such a "decomposition" useful ???

   Note: $xP_o(x^2) = 3x+7x^3+6x^5+2x^7$

1^{st} step: Divide

Evaluate the polynomial:

$P(x) = 1+3x+5x^2+7x^3+8x^4+6x^5+3x^6+2x^7$

at points $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7$ where $\omega^k=e^{i2\pi k/8}$, k = 0, 1, 2 … 7

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   Note: $xP_o(x^2)$ =

Because we can compute:

- **$P(x) = P_e(x^2)+xP_o(x^2)$**
- **$P(-x) = P_e((-x)^2)-xP_o((-x)^2) = P_e(x^2)-xP_o(x^2)$**

using only $P_e(x^2)$, $P_o(x^2)$

1^{st} step: Divide

Evaluate the polynomial:

$P(x) = 1+3x+5x^2+7x^3+8x^4+6x^5+3x^6+2x^7$

at points ω^0, ω^1, ω^2, ω^3, ω^4, ω^5, ω^6, ω^7 where $\omega^k=e^{i2\pi k/8}$, k = 0, 1, 2 … 7

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1^{st} step: Divide

Evaluate the polynomial:

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Note that the sub-problems (of the problem with size n) involve evaluation on the n/2 roots of unity !!!

1^{st} step: Divide

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Note that the sub-problems (of the problem with size n) involve evaluation on the n/2 roots of unity !!!

Recursively solve the sub-problems

Evaluate $Q(x) = 1+5x+8x^2+3x^3$ at points $\omega^k = e^{i2\pi k/4}$, k = 0, 1, 2, 3

($\omega^0 = 1$, $\omega^1 = i$, $\omega^2 = -1$, $\omega^3 = -i$)

Solve the sub-problems:

1) Evaluate $Q_e(x) = 1+8x$ at points $\omega^k = e^{i2\pi k/2}$, k = 0, 1

2) Evaluate $Q_o(x) = 5+3x$ at points $\omega^k = e^{i2\pi k/2}$, k = 0, 1

Recursively solve the sub-problems

Evaluate $Q(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, $k = 0, 1, 2, 3$

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Evaluate $Q(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, $k = 0, 1, 2, 3$

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Recursively solve the sub-problems

Evaluate $R(x) = 1+8x$ at points $\omega^k = e^{i2\pi k/2}$, $k = 0, 1$ ($\omega^0=1$, $\omega^1=-1$)

Solve the sub-problems:

1) Evaluate $R_e(x) = 1$ at point $\omega^0 = e^{i2\pi * 0/1} = 1$

2) Evaluate $R_o(x) = 8$ at point $\omega^0 = e^{i2\pi * 0/1} = 1$

Recursively solve the sub-problems

Evaluate $R(x) = 1+8x$ at points $\omega^k=e^{i2\pi k/2}$, $k = 0, 1$ ($\omega^0=1$, $\omega^1=-1$)

Solve the sub-problems:

1) Evaluate $R_e(x) = 1$ at point $\omega^0=e^{i2\pi*0/1}=1$ $R_e(1) = 1$

2) Evaluate $R_o(x) = 8$ at point $\omega^0=e^{i2\pi*0/1}=1$

Recursively solve the sub-problems

Evaluate $R(x) = 1+8x$ at points $\omega^k=e^{i2\pi k/2}$, $k = 0, 1$ ($\omega^0=1$, $\omega^1=-1$)

Solve the sub-problems:

1) Evaluate $R_e(x) = 1$ at point $\omega^0=e^{i2\pi*0/1}=1$ $R_e(1) = 1$

2) Evaluate $R_o(x) = 8$ at point $\omega^0=e^{i2\pi*0/1}=1$ $R_o(1) = 8$

Recursively solve the sub-problems

Evaluate $R(x) = 1+8x$ at points $\omega^k=e^{i2\pi k/2}$, $k = 0, 1$ ($\omega^0=1$, $\omega^1=-1$)

Solve the sub-problems:

1) Evaluate $R_e(x) = 1$ at point $\omega^0=e^{i2\pi^*0/1}=1$ $R_e(1) = 1$

2) Evaluate $R_o(x) = 8$ at point $\omega^0=e^{i2\pi^*0/1}=1$ $R_o(1) = 8$

Combine the solutions:

Recursively solve the sub-problems

Evaluate $R(x) = 1+8x$ at points $\omega^k=e^{i2\pi k/2}$, $k = 0, 1$ ($\omega^0=1$, $\omega^1=-1$)

Solve the sub-problems:

1) Evaluate $R_e(x) = 1$ at point $\omega^0=e^{i2\pi*0/1}=1$ $R_e(1) = 1$

2) Evaluate $R_o(x) = 8$ at point $\omega^0=e^{i2\pi*0/1}=1$ $R_o(1) = 8$

Combine the solutions:

$R(1) = R_e(1^2) + 1* R_o(1^2) = 1 + 8 = 9$
$R(-1) = R_e((-1)^2) - 1*R_o((-1)^2) = 1 - 8 = -7$

Recursively solve the sub-problems

Evaluate $R(x) = 1+8x$ at points $\omega^k=e^{i2\pi k2}$, $k = 0, 1$ ($\omega^0=1, \omega^1=-1$)

Solve the sub-problems:

1) Evaluate $R_e(x) = 1$ at point $\omega^0=e^{i2\pi * 0/1} =1$ $R_e(1) = 1$

2) Evaluate $R_o(x) = 8$ at point $\omega^0=e^{i2\pi * 0/1} =1$ $R_o(1) = 8$

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Observe the reuse of $R_e(1)$ and $R_o(1)$ in the combine step!!!

WHY ???

Recursively solve the sub-problems

Evaluate $R(x) = 1+8x$ at points $\omega^k=e^{i2\pi k/2}$, $k = 0, 1$ ($\omega^0=1$, $\omega^1=-1$)

Solve the sub-problems:

1) Evaluate $R_e(x) = 1$ at point $\omega^0=e^{i2\pi*0/1}=1$ $R_e(1) = 1$

2) Evaluate $R_o(x) = 8$ at point $\omega^0=e^{i2\pi*0/1}=1$ $R_o(1) = 8$

Combine the solutions:

$R(1) = R_e(1^2) + 1^* R_o(1^2) = 1 + 8 = 9$

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Observe the reuse of $R_e(1)$ and $R_o(1)$ in the combine step!!!

When we square the n roots of unity we get the n/2 roots of unity, where we have already evaluated the sub-problems !!!

Recursively solve the sub-problems

Evaluate $Q(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, $k = 0, 1, 2, 3$

($\omega^0 = 1$, $\omega^1 = i$, $\omega^2 = -1$, $\omega^3 = -i$)

Solve the sub-problems:

1) Evaluate $Q_e(x) = 1+8x$ at points $\omega^k=e^{i2\pi k/2}$, $k = 0, 1$: $Q_e(1) = 9$, $Q_e(-1) = -7$ ✔

2) Evaluate $Q_o(x) = 5+3x$ at points $\omega^k=e^{i2\pi k/2}$, $k = 0, 1$

Recursively solve the sub-problems

Evaluate $Q(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, k = 0, 1, 2, 3
($\omega^0 = 1$, $\omega^1 = i$, $\omega^2 = -1$, $\omega^3 = -i$)
Solve the sub-problems:

1) Evaluate $Q_e(x) = 1+8x$ at points $\omega^k=e^{i2\pi k/2}$, k = 0, 1 : $Q_e(1) = 9$, $Q_e(-1) = -7$

2) Evaluate $Q_o(x) = 5+3x$ at points $\omega^k=e^{i2\pi k/2}$, k = 0, 1

Similarly we get: $Q_o(1) = 8$, $Q_o(-1) = 2$

Recursively solve the sub-problems

Evaluate $Q(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, k = 0, 1, 2, 3

($\omega^0 = 1$, $\omega^1 = i$, $\omega^2 = -1$, $\omega^3 = -i$)

Solve the sub-problems:

1) Evaluate $Q_e(x) = 1+8x$ at points $\omega^k=e^{i2\pi k/2}$, k = 0, 1 : $Q_e(1) = 9$, $Q_e(-1) = -7$

2) Evaluate $Q_o(x) = 5+3x$ at points $\omega^k=e^{i2\pi k/2}$, k = 0, 1

Similarly we get: $Q_o(1) = 8$, $Q_o(-1) = 2$

Combine the solutions:

$Q(1) = Q_e(1^2) + 1 * Q_0(1^2) = 9 + 8 = 17$

$Q(-1) = Q_e((-1)^2) - 1 * Q_0((-1)^2) = 9 - 8 = 1$

$Q(i) = Q_e(i^2) + i * Q_0(i^2) = = Q_e(-1) + i * Q_0(-1) = -7 + 2i$

$Q(-i) = Q_e((-i)^2) - i * Q_0((-i)^2) = = Q_e(-1) - i * Q_0(-1) = -7 -2i$

Recursively solve the sub-problems

Evaluate $Q(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, $k = 0, 1, 2, 3$
($\omega^0 = 1$, $\omega^1 = i$, $\omega^2 = -1$, $\omega^3 = -i$)
Solve the sub-problems:

1) Evaluate $Q_e(x) = 1+8x$ at points $\omega^k=e^{i2\pi k/2}$, $k= 0, 1$: $Q_e(1) = 9$, $Q_e(-1) = -7$

2) Evaluate $Q_o(x) = 5+3x$ at points $\omega^k=e^{i2\pi k/2}$, $k = 0, 1$

Similarly we get: $Q_o(1) = 8$, $Q_o(-1) = 2$

Combine the solutions:

Observe the reuse of $Q_e(1)$, $Q_e(-1)$ and $Q_o(1)$, $Q_o(-1)$ in the combine step!!!

$Q(1) = Q_e(1^2) + 1 * Q_0(1^2) = 9 + 8 = 17$

$Q(-1) = Q_e((-1)^2) - 1 * Q_0((-1)^2) = 9 - 8 = 1$

$Q(i) = Q_e(i^2) + i * Q_0(i^2) = = Q_e(-1) + i * Q_0(-1) = -7 + 2i$

$Q(-i) = Q_e((-i)^2) - i * Q_0((-i)^2) = = Q_e(-1) - i * Q_0(-1) = -7 -2i$

Recursively solve the sub-problems

Evaluate the polynomial:

$P(x) = 1+3x+5x^2+7x^3+8x^4+6x^5+3x^6+2x^7$

at points $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7$ where $\omega^k = e^{i2\pi k/8}$, k = 0, 1, 2 … 7

Recursively solve the following problems and combine the solutions.

1) Evaluate $P_e(x) = 1+5x+8x^2+3x^3$ at points $\omega^k = e^{i2\pi k/4}$, k = 0, 1, 2, 3

   $P_e(1) = 17$, $P_e(-1) = 1$, $P_e(i) = -7 + 2i$, $P_e(-i) = -7 -2i$ ✔

2) Evaluate $P_o(x) = 3+7x+6x^2+2x^3$ at points $\omega^k = e^{i2\pi k/4}$, k = 0, 1, 2, 3

Recursively solve the sub-problems

Evaluate the polynomial:

$P(x) = 1+3x+5x^2+7x^3+8x^4+6x^5+3x^6+2x^7$

at points $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7$ where $\omega^k=e^{i2\pi k/8}$, k = 0, 1, 2 … 7

Recursively solve the following problems and combine the solutions.

1) Evaluate $P_e(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, k = 0, 1, 2, 3

   $P_e(1) = 17$, $P_e(-1) = 1$, $P_e(i) = -7 + 2i$, $P_e(-i) = -7 -2i$

2) Evaluate $P_o(x) = 3+7x+6x^2+2x^3$ at points $\omega^k=e^{i2\pi k/4}$, k = 0, 1, 2, 3

   Similarly we get:

   $P_o(1) = 18$, $P_o(-1) = 0$, $P_o(i) = -3 + 5i$, $P_o(-i) = -3 -5i$

2^{nd} step: Combine

Evaluate the polynomial:
$P(x) = 1+3x+5x^2+7x^3+8x^4+6x^5+3x^6+2x^7$
at points $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7$ where $\omega^k=e^{i2\pi k/8}$, $k = 0, 1, 2 \ldots 7$

Recursively solve the following problems and combine the solutions.

1) Evaluate $P_e(x) = 1+5x+8x^2+3x^3$ at points $\omega^k=e^{i2\pi k/4}$, $k = 0, 1, 2, 3$
   $P_e(1) = 17$, $P_e(-1) = 1$, $P_e(i) = -7 + 2i$, $P_e(-i) = -7 -2i$
2) Evaluate $P_o(x) = 3+7x+6x^2+2x^3$ at points $\omega^k=e^{i2\pi k/4}$, $k = 0, 1, 2, 3$
   $P_o(1) = 18$, $P_o(-1) = 0$, $P_o(i) = -3 + 5i$, $P_o(-i) = -3 -5i$

Putting it all (back) together

1. Divide
2. Conquer
3. **Combine**

Polynomial Evaluation

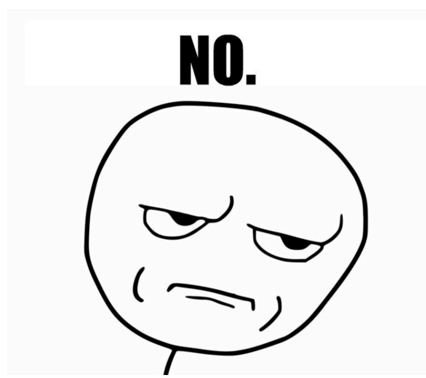
Input: $P(x) = 1 + 3x + 5x^2 + 7x^3 + 8x^4 + 6x^5 + 3x^6 + 2x^7$

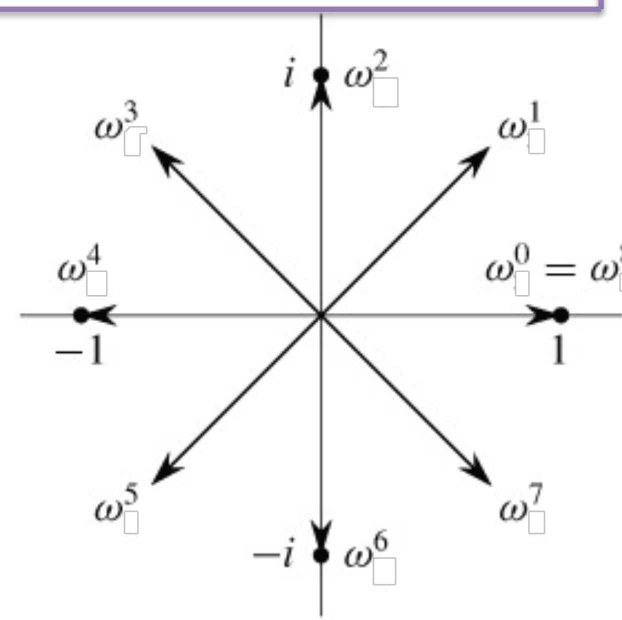
Output: $P(\omega^0), P(\omega^1), P(\omega^2), P(\omega^3), P(\omega^4), P(\omega^5), P(\omega^6), P(\omega^7)$

*which is the same as:

$$P(1), P(\omega), P(i), P(\omega^3), P(-1), P(-\omega), P(-i), P(-\omega^3)$$

Evaluate!





ok, how about just getting the subproblems?

Pull even terms

$$P(x) = 1 + 3x + 5x^2 + 7x^3 + 8x^4 + 6x^5 + 3x^6 + 2x^7$$

$$P_e(x^2) = 1 + 5x^2 + 8x^4 + 3x^6$$

Pull odd terms

$$P(x) = 1 + 3x + 5x^2 + 7x^3 + 8x^4 + 6x^5 + 3x^6 + 2x^7$$

$$P_o(x^2) = 3 + 7x^2 + 6x^4 + 2x^6$$

We've split the polynomial!

$$P(x) = P_e(x^2) + xP_o(x^2)$$

First Subproblem
Evaluate:

$$P_e(x^2) = 1 + 5x^2 + 8x^4 + 3x^6$$

For: $P(1), P(\omega), P(i), P(\omega^3), P(-1), P(-\omega), P(-i), P(-\omega^3)$

But we don't have to evaluate at each of these points – only their squares

$$P_e(1^2) = P_e((-1)^2) = P_e(1) \qquad 1$$

$$P_e(i^2) = P_e((-i)^2) = P_e(-1) \qquad -1$$

$$P_e(\omega^2) = P_e((-\omega)^2) = P_e(\omega^2) = P_e(i) \qquad i$$

$$P_e((\omega^3)^2) = P_e((-\omega^3)^2) = P_e(\omega^6) = P_e(-i) \qquad -i$$

$$P_e(x^2) = 1 + 5x^2 + 8x^4 + 3x^6$$

Polynomial Evaluation

$$\text{Let } y = x^2$$

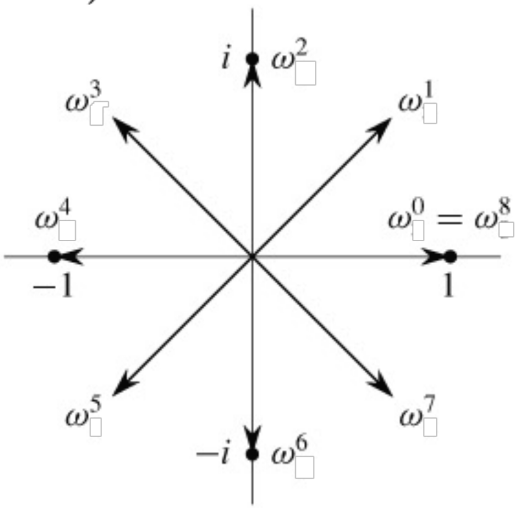
Input: $$P'(y) = 1 + 5y + 8y^2 + 3y^3$$

Output: $$P'(1), P'(i), P'(-1), P'(-i)$$

*which is the same as:

$$P'(\zeta^0), P'(\zeta^1), P'(\zeta^2), P'(\zeta^3)$$

where ζ is the 4th root of unity



But we’ve already solved this! !!!!!!!!!!!!!!!!!!!!

Polynomial Evaluation

Input: $P(x) = 1 + 3x + 5x^2 + 7x^3 + 8x^4 + 6x^5 + 3x^6 + 2x^7$

Output: $P(1), P(\omega), P(i), P(\omega^3), P(-1), P(-\omega), P(-i), P(-\omega^3)$

$P_e(x^2) = 1 + 5x^2 + 8x^4 + 3x^6$

$P_o(x^2) = 3 + 7x^2 + 6x^4 + 2x^6$

$$P_e(1) = 17$$
$$P_e(-1) = 1$$
$$P_e(i) = -7 + 2i$$
$$P_e(-i) = -7 - 2i$$

$$P_o(1) = 18$$
$$P_o(-1) = 0$$
$$P_o(i) = -3 + 5i$$
$$P_o(-i) = -3 - 5i$$

Polynomial Evaluation

Input: $P(x) = 1 + 3x + 5x^2 + 7x^3 + 8x^4 + 6x^5 + 3x^6 + 2x^7$

$$P(x) = P_e(x^2) + xP_o(x^2)$$

$P_e(x^2) = 1 + 5x^2 + 8x^4 + 3x^6$

$P_o(x^2) = 3 + 7x^2 + 6x^4 + 2x^6$

$$P_e(1) = 17$$

$$P_o(1) = 18$$

$$P_e(-1) = 1$$

$$P_o(-1) = 0$$

$$P_e(i) = -7 + 2i$$

$$P_o(i) = -3 + 5i$$

$$P_e(-i) = -7 - 2i$$

$$P_o(-i) = -3 - 5i$$

We have all the pieces, so what is:

$$P(1), P(\omega), P(i), P(\omega^3), P(-1), P(-\omega), P(-i), P(-\omega^3)$$

Polynomial Evaluation

Input: $P(x) = 1 + 3x + 5x^2 + 7x^3 + 8x^4 + 6x^5 + 3x^6 + 2x^7$

$$P_e(1) = 17 \qquad P_o(1) = 18$$

$$P_e(-1) = 1 \qquad P_o(-1) = 0$$

$$P_e(i) = -7 + 2i \qquad P_o(i) = -3 + 5i$$

$$P_e(-i) = -7 - 2i \qquad P_o(-i) = -3 - 5i$$

$$P(1) = P_e(1^2) + 1 * P_o(1^2) = P_e(1) + P_o(1) = 17 + 18 = 35$$

$$P(-1) = P_e((-1)^2) - 1 * P_o((-1)^2) = P_e(1) - P_o(1) = 17 - 18 = -1$$

$$P(i) = P_e(i^2) + i * P_o(i^2) = P_e(-1) + iP_o(-1) = 1 + i * 0 = 1$$

$$P(-i) = P_e((-i)^2) - i * P_o((-i)^2) = P_e(-1) - iP_o(-1) = 1 - i * 0 = 1$$

Polynomial Evaluation

Input: $P(x) = 1 + 3x + 5x^2 + 7x^3 + 8x^4 + 6x^5 + 3x^6 + 2x^7$

$$P_e(1) = 17$$

$$P_o(1) = 18$$

$$P_e(-1) = 1$$

$$P_o(-1) = 0$$

$$P_e(i) = -7 + 2i$$

$$P_o(i) = -3 + 5i$$

$$P_e(-i) = -7 - 2i$$

$$P_o(-i) = -3 - 5i$$

$$P(\omega^1) = P_e(\omega^2) + \omega * P_o(^2) = P_e(i) + \omega * P_o(i) = (-7 + 2i) + \omega(-3 + 5i)$$

$$P(-\omega^1) = P_e((-\omega)^2) - \omega * P_o((-\omega)^2) = P_e(i) - \omega * P_o(i) = (-7 + 2i) - \omega(-3 + 5i)$$

$$P(\omega^3) = P_e(\omega^6) + \omega^3 * P_o(\omega^6) = P_e(-i) + \omega^3 * P_o(-i) = (-7 - 2i) + \omega^3(-3 - 5i)$$

$$P(-\omega^3) = P_e((-\omega^3)^2) - \omega^3 * P_o((-\omega^3)^2) = P_e(-i) - \omega^3 * P_o(-i) = (-7 - 2i) - \omega^3(-3 - 5i)$$

Polynomial Evaluation

Input: $P(x) = 1 + 3x + 5x^2 + 7x^3 + 8x^4 + 6x^5 + 3x^6 + 2x^7$

$$P(\omega^0) = 35$$
$$P(\omega^1) = -7 + 2i + \omega(-3 + 5i)$$
$$P(\omega^2) = 1$$
$$P(\omega^3) = -7 - 2i + \omega^3(-3 - 5i)$$
$$P(\omega^4) = -1$$
$$P(\omega^5) = -7 + 2i - \omega(-3 + 5i)$$
$$P(\omega^6) = 1$$
$$P(\omega^7) = -7 - 2i - \omega^3(-3 - 5i)$$