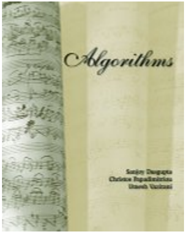
CS 170: Algorithms



Prof David Wagner.
Slides edited from a version created by Prof. Satish Rao.
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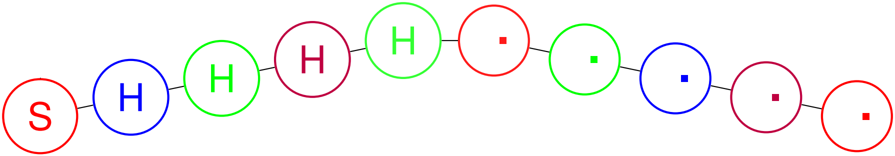
CS 170: Algorithms



Cycle in a directed graph?

Fast algorithm for finding out whether directed graph has cycle?

For each edge (u, v) remove, check if v is connected to u
$O(|E|(|E| + |V|))$.

Linear Time (i.e. $O(|V| + |E|)$)?

Directed graphs.

$G = (V, E)$

vertices V.

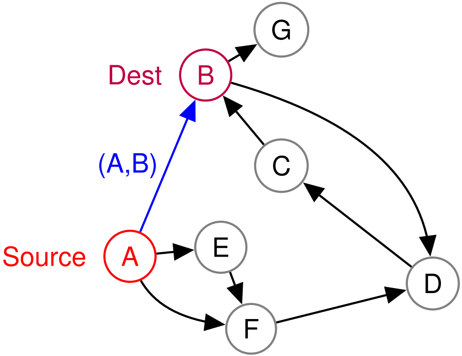
edges $E \subseteq V \times V$.

Edge: (u, v)

From u to v.

Source – u

Dest – v



Introspection: pre/post.

Previsit(v):

1. Set pre[v] := clock.
2. clock := clock+1

Postvisit(v):

1. Set post[v] := clock.
2. clock := clock+1

DFS(G):

0. Set clock := 0.

$\vdots$

Clock: goes up to 2 times number of tree edges.
First pre: 0

Property: For any two nodes, u and v, $[pre(u), post(u)]$ and $[pre(v), post(v)]$ are either disjoint or one is contained in other.

Interval is "clock interval on stack."

Either both on stack at some point (contained) or not (disjoint.)

Directed Acyclic Graphs: Depth First Search

Edge: (u, v)

From u to v.

Source – u

Dest – v

Given a DFS forest, the edge (u, v) of the graph is a

Tree edge – "Direct call tree of explore", $(u, v) \in T$,
$pre(u) < pre(v) < post(v) < post(u)$.

Forward edge – "Edge to descendant (not in tree), $(u, v) \notin T$,
$pre(u) < pre(v) < post(v) < post(u)$

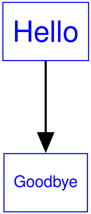
Back edge – "Edge to ancestor" $(u, v) \notin T$,
$pre(v) < pre(u) < post(u) < post(v)$

Cross edge – None of the above: $(u, v) \notin T$,
$pre(v) < post(v) < pre(u) < post(u)$
v already explored before u is visited.

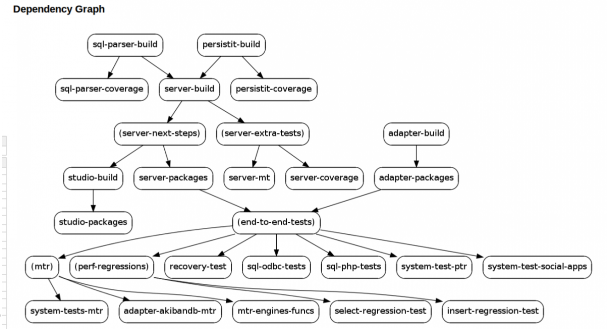
These are all the possible edges.

Directed Acyclic Graph

Directed Graph ...without cycles.
Why?

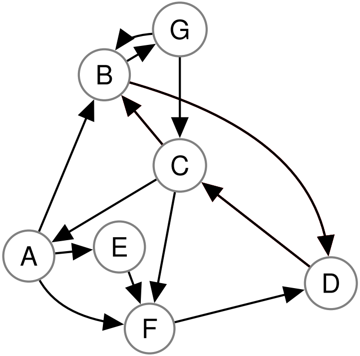


“Hello” before “Goodbye

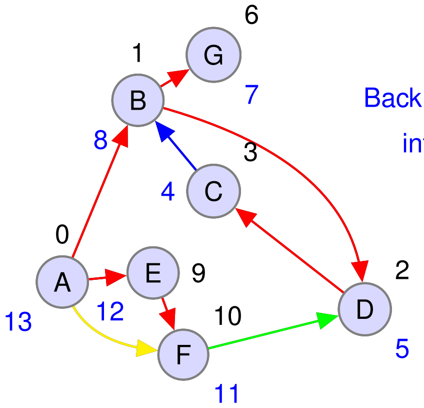


Example.

Acyclic Graph?



Depth first search: directed.



Back edge (u, v): int(v) contains int(u).

int$(C) = [3, 4]$ and int$(B) = [1, 8]$.

Back edge (u, v)
....edge to ancestor
..........path of tree edges from v to u.
Back edge means cycle! $\implies$ not acyclic!

Testing for cycle.

Thm: A graph has a cycle if and only if there is a back edge in any DFS.

Proof:
We just saw: Back edge $\implies$ cycle!

In the other direction: Assume there is a cycle

$v_0 \to v_1 \to v_2 \cdots \to v_k \to v_0$

Assume that v_0 is the first node explored in the cycle
(without loss of generality since can renumber vertices.)

When **explore**(v_0) returns all nodes on cycle explored.

All int[v_i] in int[v_0]!

$\implies (v_k, v_0)$ is a back edge.

Cycle $\implies$ back edge! $\square$

Fast checking algorithm.

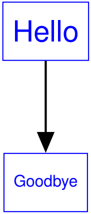
Thm: A graph has a cycle if and only if there is back edge.
Algorithm ??

Run DFS.
$O(|V|+|E|)$ time.

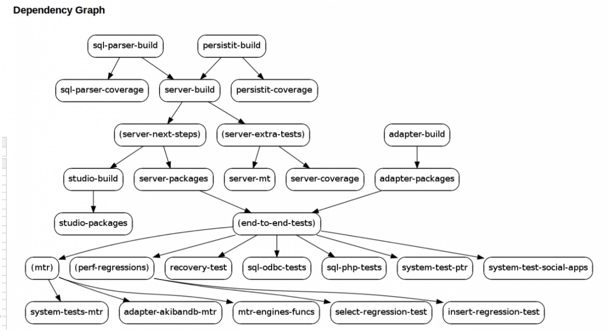
For each edge (u,v): is int(u) in int(v).
$O(|E|)$ time.

$O(|V|+|E|)$ time algorithm for checking if graph is acyclic!

Directed Acyclic Graph



“Hello” before “Goodbye”

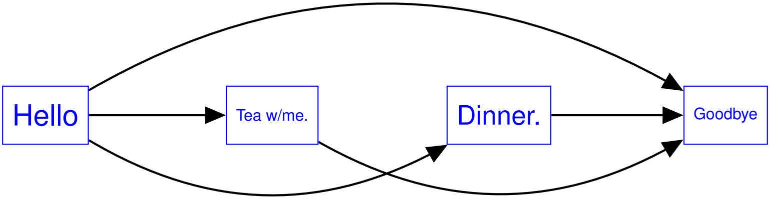


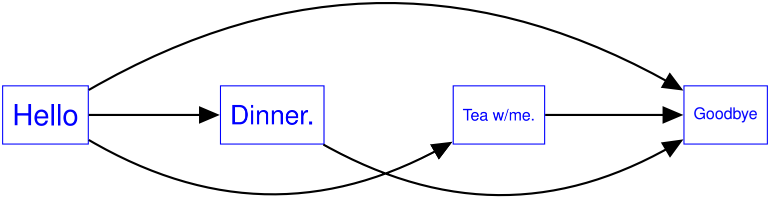
No cycles! Can tell in linear time!

Ohhh...Kayyyy...
Really want to find ordering for build!

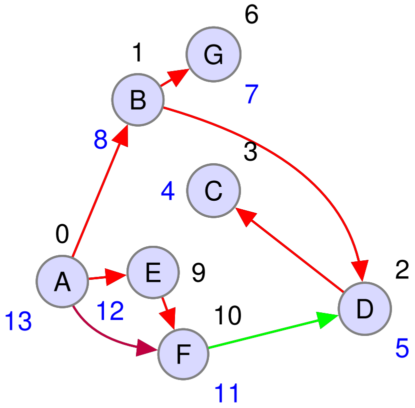
Linearize.

Topological Sort: For $G = (V, E)$, find ordering of all vertices where each edge goes from earlier vertex to later in acyclic graph.





Topological Sort Example.



A linear order:

A, E, F, B, G, D, C

In DFS: When is A popped off stack?

Last! When is E popped off? second to last. ...